A Fast and Flexible Algorithm for Total Variation Regularization

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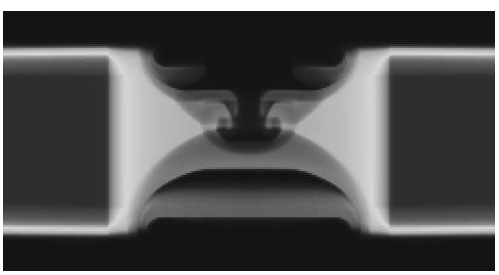
Total Variation (TV) regularization has become a very popular method for a wide variety of image restoration problems, including denoising, deconvolution, and tomography [1, 2, 3]. Given data f and a forward problem represented by linear operator K, the standard ℓ^2-TV regularized solution is the minimum of the functional

$$T(u) = \frac{1}{2}\int_\Omega |Ku - f|^2 dx + \lambda \int_\Omega |\nabla u| dx,$$

where the first term is the *data fidelity* term which ensures that there is approximate correspondence between solution u and data f, the second term is the *regularization* term which penalizes solutions containing large jumps (which often correspond to noise artifacts), and λ is a weighting factor which balances the competing requirements of the two terms.



Original image

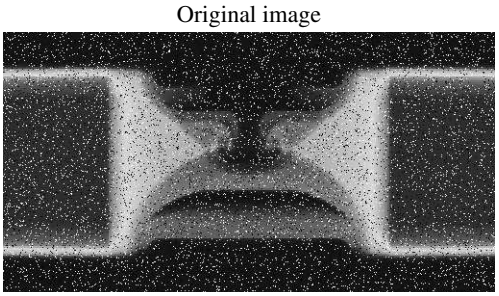
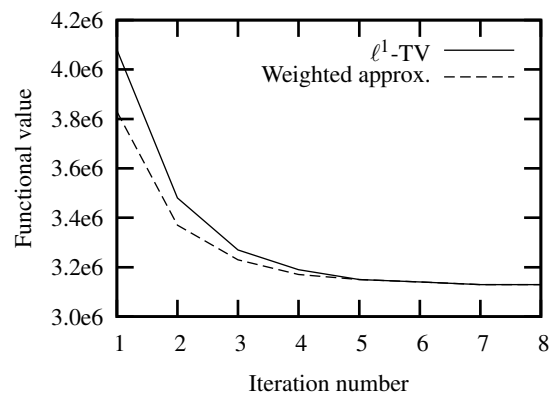


Image with 10% shot noise. SNR: 1.1db

Recently, a number of researchers have begun to consider modifications to this functional, with particular interest on ℓ^1-TV [4]

$$T(u) = \int_\Omega |Ku - f| dx + \lambda \int_\Omega |\nabla u| dx,$$

which has a number of advantages over the standard ℓ^2-TV, including superior performance with non-Gaussian noise such as salt and pepper noise [4], and applications including cDNA microarray image processing [5], and illumination normalization. While efficient algorithms are available for solving ℓ^2-TV problems, the development of effective methods for computing solutions to ℓ^1-TV problems has lagged behind the advances in theoretical understanding of their properties and advantages.



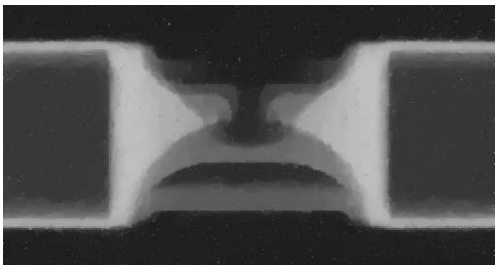
Convergence of weighted approximation to ℓ^1-TV functional.

We have developed a new algorithm for solving a generalized TV functional of the form

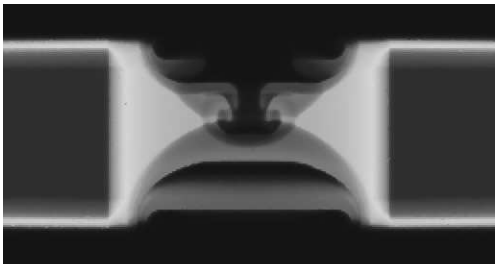
$$T(u) = \frac{1}{p}\int_\Omega |Ku - f|^p dx + \frac{\lambda}{q}\int_\Omega |\nabla u|^q dx,$$

which includes both the standard ℓ^2-TV and the recent ℓ^1-TV as special cases. This algorithm, closely related to the Iteratively Reweighted Least Squares (IRLS) [6] method, represents the norms in the data fidelity and regularization terms, which are difficult to minimize when $p, q \neq 2$, by weighted versions of ℓ^2 norms, which are easily minimized. At each iteration of the algorithm,

these weights are updated to obtain the desired norms. In addition to its flexibility, allowing it to solve a TV functional tuned for specific types of data and noise models, this method also provides a very fast algorithm for the l^1-TV formulation, comparable to the state of the art for the denoising problem (when operator K is the identity), and significantly faster than any other algorithm of which we are aware for more general problems, such as deconvolution.



Denoised using ℓ^2 TV. SNR: 12.8db



Denoised using ℓ^1 TV. SNR: 31.4db

Our work has been presented at a recent conference [7], and a journal paper describing the method is in preparation. A software implementation [8] is available under an open-source license.

Acknowledgements

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References

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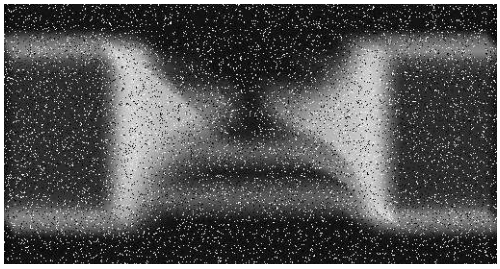
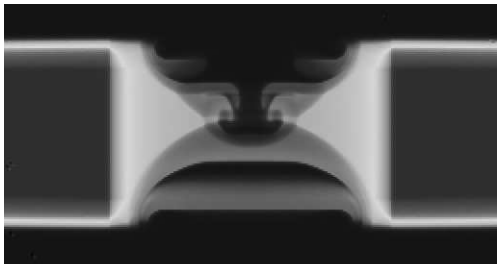


Image with motion blur and 10% shot noise. SNR: 0.7db



Deconvolved and denoised using ℓ^1 TV. SNR: 33.0db

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