Node Reconnection Algorithm for Mimetic Finite Difference Discretizations of Elliptic Equations

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Most efficient adaptive mesh methods employ only a few strategies, including local mesh refinement (h-adaptation), movement of mesh nodes (r-adaptation), and node reconnection (c-adaptation). Despite of its simplicity, node reconnection is the least popular of the three. However, using only node reconnection, the discretization error can be significantly reduced [1]. We developed and numerically analyzed a new c-adaptation algorithm for mimetic finite difference discretizations of elliptic equations on triangular meshes.

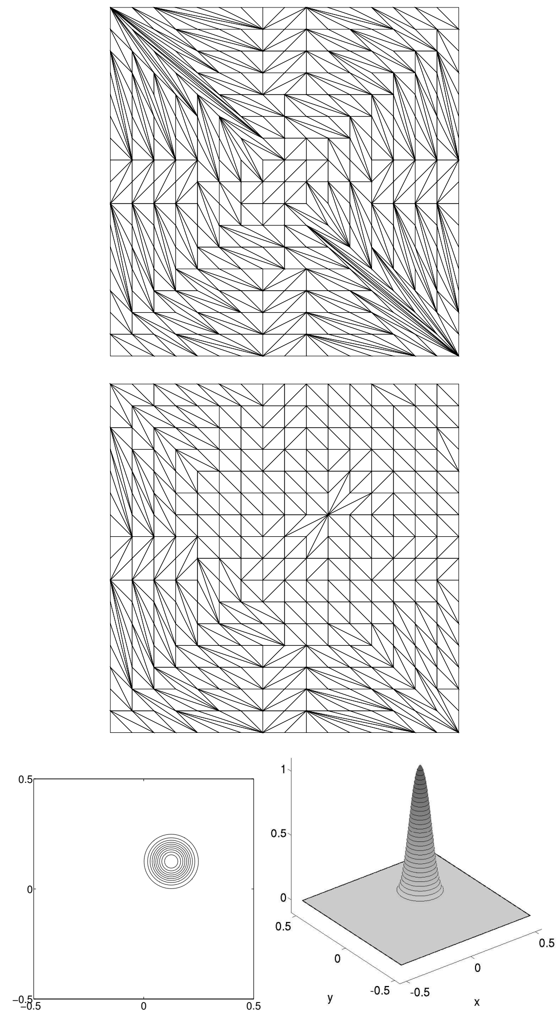
Let us consider the model elliptic boundary value problem

$$\text{div}\,\vec{J} = q, \qquad \vec{J} = -K \text{ grad } \phi$$

where ϕ is a scalar function referred to as intensity, $\vec{J}$ is a vector function, K denotes a full symmetric diffusion tensor, and q is a source function. The problem is posed in a polygonal domain with Dirichlet boundary conditions. It is discretized using the mimetic finite difference method, which mimics the fundamental properties of the continuous equation [2].

The reconnection strategy requires an efficient error indicator, which is used to decide where to modify the mesh. We propose a new error indicator which is well suited for mimetic finite difference discretizations. On each triangular element, a linear function is reconstructed using a least-squares fit to the discrete solution. The error is then estimated by integrating the square of the discontinuity in the linear functions across triangle edges. Note that this error indicator can also be used on unstructured polygonal meshes.

The c-adaptation strategy on triangular meshes can simply be described as a sequence of edge



Adaptation of a bad quality mesh on the domain $[-0.5,0.5]^2$ *for solution of the Dirichlet problem with intensity given by* $\phi(x,y) = 1 - \tanh\left(|(x,y)-(0.125,0.125)|^2/0.01\right)$. *Global* L_2 *error on the adapted mesh (center) was decreased by 71% w.r.t. the original (top). Note, that swapping was not necessary where the intensity function (bottom) is flat, i.e. far from the peak shown by isolines and surface plot* swaps. In particular, each interior edge has two adjacent triangles that form a quadrilateral patch for which this edge is diagonal. Edge swapping means deleting this edge and introducing a new edge which coincides with the other diagonal of the patch. In our mesh adaptation process, we loop over all interior edges and swap them if the mesh geometry allows it and the estimate of discretization error is reduced. For more details on the entire process, see [3].

Numerical results show that even the change of a few edges based on the proposed error indicator can greatly improve the solution. Combination of the node reconnection technique with other mesh adaptation strategies is a promising and challenging task for future research.

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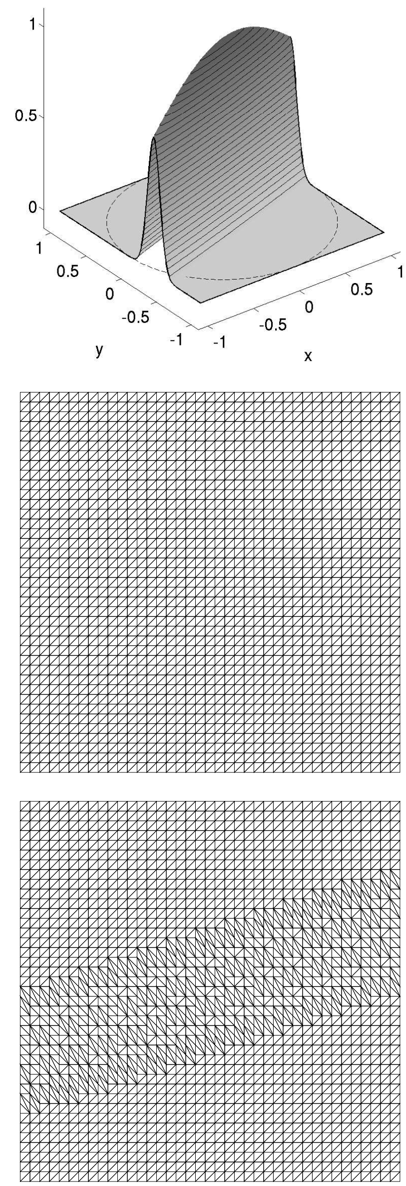
References

[1] P. Váchal and R. V. Garimella. On quality improvement of triangular meshes using node reconnection. In A. Handlovičová, Z. Krivá, K. Mikula, and D. Ševčovič, editors, *Proceedings of Algoritmy 2005, 17th Conference on Scientific Computing*, pages 93–102, Bratislava, Slovakia, 2005. Slovak University of Technology.

[2] V. Ganzha, R. Liska, M. Shashkov, and C. Zenger. Support operator method for Laplace equation on unstructured triangular grid. *Selcuk Journal of Applied Mathematics*, 3(1):21–48, 2002.

[3] M. Berndt, K. Lipnikov, P. Váchal, and M. Shashkov. A node reconnection algorithm for mimetic finite difference discretizations of elliptic equations on triangular meshes. Technical Report LA-UR-05-4620, Los Alamos National Laboratory, 2005. Submitted to Comm. Math. Sci.



Adaptation of a regular mesh on the domain $[-1,1]^2$ *for solution of the Dirichlet problem with* $\phi(x,y) = \exp\{-a^2[(x+by)^2 + c^2(x-b/y)^2]\}$, *where* $a = 1/2$, $b = 1/3$ *and* $c = 16/3$. *After two rounds of over all interior edges, the* L_2 *discretization error decreased by 65%. Top: Analytical intensity function* $\phi(x,y)$, *center: original mesh, bottom: adapted mesh after two iterations*