Predicting the Small Scales of α-Models for Turbulence

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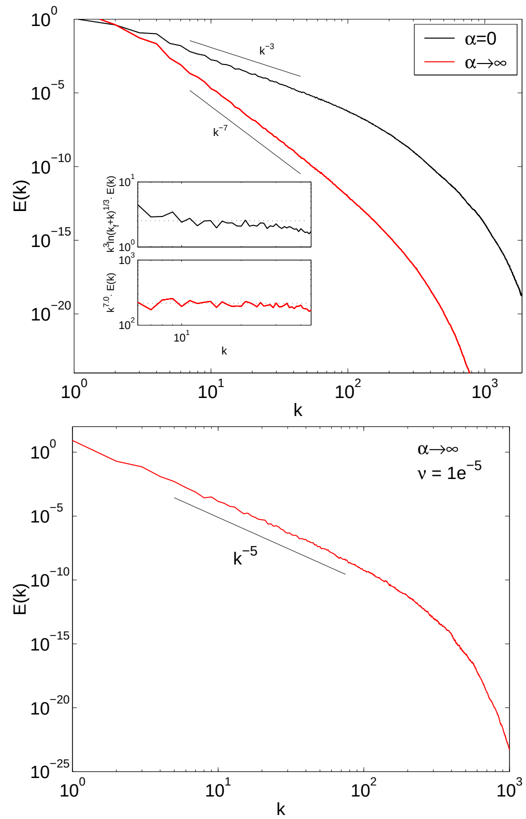
The use of numerical models for the Navier-Stokes equations of fluid dynamics is an established practice in the study of turbulent flows. The calculation of flow from a model, instead of the underlying Navier-Stokes equations, allows for the use of less computing resources for a given flow. The so-called 'α-models' for turbulence typically use a smoothed velocity field to transport a rough velocity field. The smoothing is performed over a length scale α. The expectation is that the smoothed field recovers the large-scale (greater than α) statistical properties, such as the energy spectrum, of the 'true' turbulent flow which is not smooth. However, for scales smaller than α the statistics are not easily deduced analytically since there are two participating 'velocities' which have different characteristic timescales and presumably different dynamics. In this work we use numerical simulations to arrive at an empirical hypothesis as to how one can predict the scaling of the energy spectrum of an α model. We have thus deduced that the ambiguity in choice of dynamical timescales in α-models, due to the presence of two velocities, disappears when one simply considers the relevant conserved quantity and uses its timescale to govern the evolution of all other quantities of interest, including the energy.

Suppose we denote the smoothed velocity by u, and the rough (unsmooothed) velocity by v. Is it u, v or some combination of the two which determines the statistics of scales smaller than α? In this work we examine this question numerically using two different α-models, the Navier-Stokes-α (NS-α) model and the Leray-α model. We study the energy spectra of these two models



Spectra from 4096^2 simulations of the Navier-Stokes-α and Leray-α models. Top panel: Black curve – energy spectrum of the $\alpha = 0$ (Navier-Stokes) equations, showing close to k^{-3} scaling in the enstrophy cascade range $5 < k < 50$, as expected. Red curve – energy spectrum of the $NS - \alpha$ model simulation for $\alpha \to \infty$, scales as k^{-7}, consistent with dynamics governed by the flux of the converved enstrophy in this regime. Inset – compensated spectra. Bottom panel: Energy spectrum for a 4096^2 simulation of the Leray-al pha model for $\alpha \to \infty$. The scaling is k^{-5} in the wavenumber region $5 < k < 50$, consistent with dynamics governed by the flux of the converved enstrophy, different from that of Navier-Stokes-α, in this regime.

in the two-dimensional case in the limit as $\alpha \to \infty$. We will call the models respectively $NS - \infty$ and $Leray - \infty$. In this limit, *all* scales are smaller than α thus maximally extending the regime of interest. The power-law scaling of the energy spectrum may be conveniently related to the governing dynamics using simple dimensional analysis. By measuring the scaling exponent of the energy spectrum, we can thus infer which quantity governs the development of the sub-α scales. Based on the results of our simulations, we present the hypothesis that the statistics of the small scales of any α model must be determined by the dominant (conserved) quantity that is transferred downscale in the regime of scales smaller than α. The two models we study exhibit different conserved integrals, and measurement of the power-scaling of their spectra, confirms our hypothesis. This report is a summary of work published in [3, 4]

The simulations were performed at increasing resolutions up to a maximum of 4096^2 data points. The conserved quantity for NS-α is the integral (or total) of the quantity $\frac{1}{2}|\nabla \times v|^2$ known as the *enstrophy*. The scaling exponent of the energy spectrum is k^{-7} as shown in figure 1. The exponent -7 may be shown to be the only possibility if the dynamics is governed by the characteristic timescale of the flux of the conserved quantity. The conserved quantity for Leray-α is the integral of the quantity $\frac{1}{2}|\nabla \times u||\nabla \times v|$. Figure 2 shows that the spectrum for this model scales as k^{-5} which may be shown to be the only possibility for a dynamics governed by the timescale for the flux of the conserved quantity.

Thus, the results from resolved simulations of two different α-models, with different conserved quantities, support our hypothesis that the scaling of the spectrum should be governed by the characteristic timescale for the flux of the conserved quantity for the particular α model.

Acknowledgements

Los Alamos Report LA-UR-07-5768. Funded by the Department of Energy at Los Alamos National Laboratory under contracts DE-AC52-06NA25396 and the DOE Office of Science Advanced Computing Research (ASCR) program in Applied Mathematical Sciences.

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[4] E. Lunasin, E.S. Titi, and S. Kurien Spectral scaling of α-models in two-dimensional turbulence To be submitted to *Journal of Physics* (2007).