Universally embedded spherical symmetry in turbulent flows

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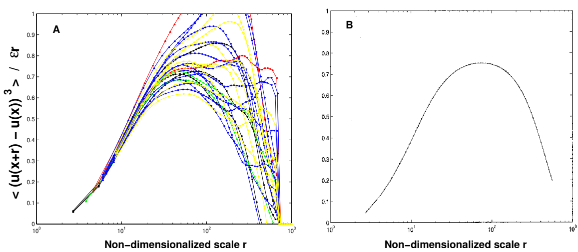
The word turbulence invokes an image of violent, agitated, incoherent motion at all scales and in all parts of a fluid. On the surface, it seems clear that the flow in river-rapids is different from a tornado which in turn is different from the controlled flow in a laboratory wind-tunnel. Is it therefore reasonable to expect that a universally quantifiable "turbulence" phenomenon exists irrespective of how the flow is generated?



A: Each colored line of connected dots is the non-dimensionalized third-order longitudinal structure function measured along a particular direction in one of our simulated flows. The lines are dramatically different from each other, especially at the large scales, which tells us the degree of asymmetry is very pronounced. B: The solid line is the same statistic, averaged over many directions in the "shaken" flow. The dotted line is the averaged quantity for the "stirred" flow. They are practically identical at all scales.

In 1941, A.N. Kolmogorov postulated that at sufficiently high Reynolds numbers, the statistics of the so-called "inertial range" of spatial scales which are much smaller than the large scales (which are usually highly anisotropic and non-universal) and much larger than the dissipative scales (where the fluid viscosity begins to play a role), have universally isotropic behavior. Based on this and a few other physically plausible, though unproven hypotheses, Kolmogorov derived the exact statistical law for third-order two-point statistics – the so-called "4/5ths-law" [1]

$$
\begin{aligned}
\langle(\delta u_L(\mathbf{r},\mathbf{x}))^3\rangle &= -\frac{4}{5}\varepsilon r \qquad (1)\\
\delta u_L(\mathbf{r},\mathbf{x}) &= [\mathbf{u}(\mathbf{x}+\mathbf{r})-\mathbf{u}(\mathbf{x})]\cdot\hat{\mathbf{r}}\\
\hat{\mathbf{r}} &= \mathbf{r}/r
\end{aligned}
$$

where $\langle\cdot\rangle$ denotes ensemble or long-time averages. The length scale r must lie in the inertial range. The lefthand side of Eq. 1 is the well-known third-order longitudinal structure function and is a measure of the flux of energy through scales of size r. The mean energy dissipation rate of the flow, computed from ensemble or long-time averages, is given by ε. The 4/5ths law is one of the few exact, non-trivial results known in the theory of statistical hydrodynamics.

A local version of the 4/5ths law was recently derived by G.L. Eyink [2]. The statement is that the K41 4/5ths law holds *instantaneously* in any chosen region of a high-Reynolds number flow if i)r is less than the size of that region and ii)the mean dissipation rate is computed over the said region and iii)the velocity differences in 1 are averaged over all angles of the sphere of radius r. This version of the K41 result does not require isotropy or homogeneity of the flow. The region considered may be of any size as long as the scale r is chosen to be smaller than it. Long-time or ensemble averages are also not required as in the original K41 theory [3]. The Eyink [2] version of K41 is truly local in space and time.

We are motivated in the present work by the existence of isotropic statistics embedded in anisotropic data as suggested by the work [2] described above. We performed fully resolved simulations of two different anisotropic flows in a periodic box of 512 grid points to a side. The large-scale anisotropy was generated by forcing each of our simulations in such a manner as to generate very different coherent shapes in the large scales. One flow had large scales which were allowed to change spatial orientation and intensity very rapidly – rather like the behavior of liquid being thoroughly shaken in a cocktail shaker. The other flow had long-lived large-scale structures with very slowly changing orientations – this would be something like the swirling structures seen in a steadily stirred cocktail. After the flows achieved statistical equilibrium, their small-scale Reynolds numbers were approximately the same value. We measured the third-order longitudinal structure function as a function of scale size r in 73 different directions in the flows. We first observed, not surprisingly, that the large-scale asymmetries in each case caused the small scale statistics of the inertial range to depend strongly on the direction along which they were measured (see Fig 1A). Furthermore, the different large scale symmetries imposed on the two flows resulted in them acquiring different small scale symmetries. In short, the two flows looked rather different from each other for any particular choice of measurement direction.

We developed a new technique which allows us to approximate the full spherical average of the local 4/5th law [2] by averaging the statistics over arbitrarily many directions. We use a method of taking the average over angles which avoids the expense and effort of interpolating the square-grid data over spherical shells, which was how this procedure has been attempted in the past. Remarkably, we saw that the statistics of both flows, averaged in this way, converged to practically indistinguishable values at *all* scales (see Fig 1B).

Our diagnosis showed that irrespective of the degree or type of asymmetry and disorder in the large scales, there is a universal, spherically symmetric component to turbulent flows. In particular, if two flows have nearly the same small-scale Reynolds number, and the same geometry, they also have *identical* underlying isotropic component at *all* scales, at least as far as the third-order energy statistics are concerned [4].

We are presently advancing this work by using our averaging technique to study the helical, parity-violating statistics of flows and much progress has been made in this direction [6]. The method is also general enough that it may be used to extract isotropic contributions to statistics of arbitrary order both in turbulence and quite possibly in other nonlinear physical systems as well.

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