

## The Impact of Mosquito-Bird Interaction on the Spread of West Nile Virus to Human Populations

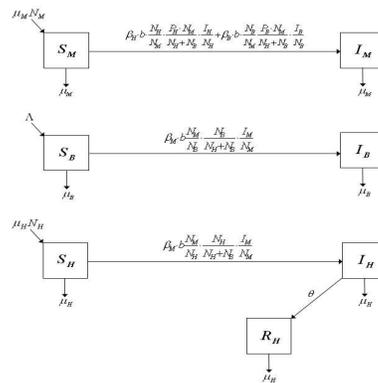
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West Nile Virus (WNV) is a rapidly spreading and potentially fatal disease. Mosquitoes spread WNV and while they feed on (and infect) mainly birds, humans are a secondary host. We establish the existence of possible multiple endemic equilibria. The possibility of multiple equilibria when  $R_0 < 1$  (backward bifurcations) implies that standard control measures are likely to be inadequate to control an epidemic. The sensitivity of  $R_0$  to parameters is studied.

While Most human infections are asymptomatic, possible symptoms range from headaches to meningitis [2]. In 1999, the first case of WNV in North America was identified in New York City. It has since spread to 46 states and infected over 9,800 people in 2003 alone [2]. In 2000, there were 71,332 birds infected in the state of New York, including 17,571 (24.6 %) American

crows and WNV is linked to high bird mortality [1].

Our model is based on a coupled non-linear system of ODE's that includes an *S-I* (Susceptible-Infected) model for the mosquito population, a *S-I-R* (Susceptible-Infected-Recovered) for the human population since most humans recover from WNV, and a *S-I* (Susceptible-Infected) model for the crow population. All populations are constant except for the crow population that exhibits a high disease induced mortality rate.



Model for WNV infection among mosquitos, birds and humans.

Figure [1] is a flowchart that outlines our model where the subscripts M,B and H denote mosquito, bird and human, respectively. S,I,R and N denote the susceptible, infected, recovered and total populations, respectively.  $\beta_H$ ,  $\beta_B$  and  $\beta_M$  are the transmission forces from humans to mosquitoes, birds to mosquitoes and mosquitoes to birds or humans, respectively.  $\Lambda$  is the bird recruitment rate,  $\mu$  is the natural per capita mortality rate,  $P_H$  and  $P_B$  are the mosquito preference probability to biting humans or birds,

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respectively.  $\theta$  is the recover rate and  $b$  is the average number of bites per mosquito per day.

The basic reproductive number ( $R_0$ ) represents the average number of secondary cases caused by a typical infectious individual in a mostly susceptible population. We introduce the following quantities:

$$N_B^* \equiv \frac{\Lambda}{\mu_B}$$

$$R_0^{MH} \equiv \frac{bN_H}{N_B^* + N_H} \frac{\beta_M}{\mu_M}$$

$$R_0^{HM} \equiv \frac{bP_H N_M}{N_B^* + N_H} \frac{\beta_H}{\mu_H + \theta}$$

$$R_0^{MB} \equiv \frac{bN_B^*}{N_B^* + N_H} \frac{\beta_M}{\mu_M}$$

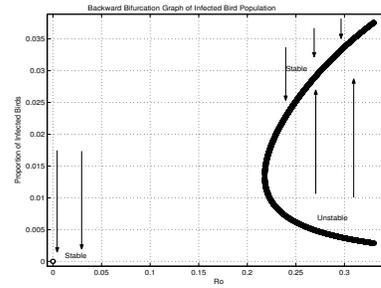
$$R_0^{BM} \equiv \frac{bP_B N_M}{N_B^* + N_H} \frac{\beta_B}{\mu_B}$$

$R_0^{ij}$  represents the secondary infections generated by  $i$  in  $j$ .  $\sqrt{R_0^H} \equiv \sqrt{R_0^{MH} \cdot R_0^{HM}}$  represents secondary infections generated by humans (or mosquitoes) in humans (mosquitoes) in a disease-free human-mosquito system and  $\sqrt{R_0^B} \equiv \sqrt{R_0^{MB} \cdot R_0^{BM}}$  represents the secondary infections generated by birds (or mosquitoes) in birds (mosquitoes) in a disease-free bird-mosquito system. Then the basic reproductive number ( $R_0$ ) of the overall system is given by:

$$R_0 = \sqrt{R_0^H + R_0^B}$$

Analysis of the equilibria reveal a disease free equilibrium, an endemic equilibrium with  $R_0 > 1$  and multiple equilibria with  $R_0 < 1$ . For detailed proof and sensitivity analysis, please contact the authors.

In Figure [2] we see that a backward bifurcation occurs in the bird population. Backward bifurcations are possible in all populations and is an indicator that the system is highly sensitive to initial conditions. In particular, it implies that even though we may



Backward bifurcation for birds varying  $\beta_B$

have excellent mosquito and disease control, an outbreak can still occur if a large number of infected birds migrates in. Further, once established, the infection is very difficult to control.

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