

## Modeling the spread of malaria

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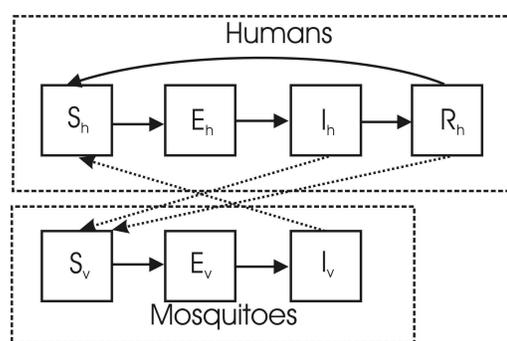
We are developing mathematical models to better understand the spread of malaria. We model the disease through ordinary differential equations where humans and mosquitoes interact and infect each other. We want to use this model to determine which factors are most responsible for the spread of malaria.

Malaria is an infectious disease caused by a parasite (*Plasmodium*) and transmitted between humans by the bites of mosquitoes (female *Anopheles*). It kills about 700,000 – 2.7 million people a year, 75% of which are African children. An estimated 40% of the world's population lives in malaria endemic areas. Over the last few years, with increasing parasite drug-resistance and mosquito insecticide-resistance, malaria has been making a comeback in many parts of the world.

Mathematical models for malaria have existed since the time of Ross (1911) [1]. We are working with a model derived from one by Ngwa and Shu [2].

The model divides the human population into 4 classes: susceptible, exposed, infected and recovered (immune). People enter the susceptible class, either through birth or migration. When an infected mosquito bites a susceptible human, there is some finite probability that the parasite (in the form of sporozoites) will be passed on to the human and the person will move to the exposed class. The parasite then travels to the liver where it develops into its next life stage. After a certain period of time, the parasite (in the form of merozoites) enters the blood stream, usually signaling the clinical onset of malaria. In our model, the people from the exposed class enter the infectious class at a rate that is the reciprocal of the duration of the latent period. After some time, the

infected humans recover and move to the recovered class. The recovered humans have some immunity to the disease and do not get clinically ill, although they still harbour low levels of parasite in their blood stream and can pass the infection to mosquitoes. After some period of time, they lose their immunity and return to the susceptible class. Humans leave the population through an outward migration rate and a natural death rate, both of which are density-dependent, and through a disease-induced death rate.



*Schematic of the model derived from Ngwa and Shu [2]. Susceptible humans get infected after contacts with infected mosquitoes. They then pass through the exposed, infected and recovered classes before reentering the susceptible population. Susceptible mosquitoes get infected through contacts with infected and recovered humans. They then pass through the exposed and infected classes. Birth into the susceptible classes and death out of all classes are not shown.*

Female mosquitoes (we do not include male mosquitoes in our model) enter the susceptible class through birth. The parasite (in the form of gametocytes) enters the mosquito, at some probability, when the mosquito bites an infected human or a recovered human (the probability of infection from a recovered human is much lower than that from an infected human); and the mosquito moves from the susceptible to the exposed class. After some period of time, dependent on the ambient temperature and humidity, the parasite develops into sporozoites and enters the mosquito's

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salivary glands; and the mosquito moves from the exposed class to the infected class. The mosquito remains infectious for life. Mosquitoes leave the population through a density-dependent death rate.

We have compiled a list of parameter values for our model from various sources of data in malaria literature. We will numerically simulate this model for parameter values corresponding to various environmental conditions. We will also analyse our model to find endemic states and to find the sensitivity of the endemic states to the parameters. This will allow us to compare the effectiveness of various control strategies, as each strategy will increase or decrease particular parameters.

We then add environmental effects by including the dependence of parameter values to temperature, rainfall and humidity. We believe that seasonal changes in the environmental conditions will result in a periodically forced model. We will analyse this model to determine the effect on malaria prevalence and incidence of seasonal changes. Also, in a similar fashion to the autonomous model, we determine the endemic state and its sensitivity to the parameters to compare various control strategies.

Malaria affects some of the poorest regions of the world, with very limited resources. Our goal with this model is to compare some of the control strategies used against malaria today and some of the ideas for future control to determine the most effective ways of reducing malarial transmission, morbidity and mortality.

## References

- [1] R. ROSS. *The prevention of malaria*. John Murray, London, 1911.
- [2] G. A. NGWA AND W. S. SHU. A mathematical model for endemic malaria with variable human and mosquito populations. *Mathematical and Computer Modelling*, 32:747–763, 2000.