Stream depletion by groundwater pumping from leaky aquifers

Vitaly A. Zlotnik
Department of Geosciences, University of Nebraska, vzlotnik@unl.edu

Daniel M. Tartakovsky
Theoretical Division, Los Alamos National Laboratory, dmt@lanl.gov

Abstract

We derive a solution for drawdown and stream depletion for shallow aquifer penetration by a stream and hydraulic aquifer connection with the underlying source bed. This solution unifies and extends the results obtained by Theis, Glover and Balmer, Hantush and Jacob, Hantush, and Hunt. We show that both hydraulic stream-aquifer connection and hydrostratigraphic conditions determine Maximum Stream Depletion Rate, which is defined as a maximum fraction of the pumping rate supplied by the stream depletion.

Introduction

Studies of stream depletion or stream depletion rate (SDR) have primarily focused on hydraulic connection between a stream and an aquifer for pumping wells in alluvial valleys (Theis, 1941, Glover and Balmer, 1954, Hantush, 1965, Hunt, 1999, Zlotnik et al., 1999, Butler et al., 2001). Solution for more complex hydrostratigraphic conditions (leaky aquifers) were mentioned by Hantush (1955, 1964), but only recently such conditions received attention in connection to the Maximum Stream Depletion Rate (MSDR) concept (Zlotnik, 2004). The MSDR is defined as a maximum fraction of the pumping rate supplied by stream depletion. This characteristic is of paramount importance for water resources managers for water balance assessment and adjudication of water rights. Zlotnik (2004) showed that this fraction could vary from 0 to 1. However, simplified models based on the Hantush (1955, 1964) approach assume perfect connection between the stream ant the alluvial aquifer, thereby providing an upper limit of SDR. In many cases, an overestimating the stream depletion is undesirable in developing a stream water budget. Therefore, there is need for a model that takes into account both the stream-aquifer connection and leaky conditions.

Objectives of this article are to present a solution that takes into account shallow aquifer penetration by a stream and hydraulic aquifer connection with the underlying source bed. We show quantitatively that both hydraulic stream-aquifer connection and hydrostratigraphic conditions determine MSDR. At various limits, our solution reduces to the classic Theis, Glover and Balmer, Hantush and Jacob, and Hantush solutions, as well as to a more recent solution by Hunt (1999).

Problem Formulation

Consider a well operating with a constant pumping rate Q in a leaky aquifer at a distance l from a shallow stream. A schematic cross section of our model and relevant parameters are shown in Figure 1. Our assumptions for problem formulation are as follows:

- The Dupuit assumptions are valid, and hydraulic head $h(x, y, t)$ is a function of Cartesian coordinates x and y and time t,
- Alluvial aquifer is of infinite extent and hydraulic conductivity k and storativity S are homogeneous and isotropic,
- Relative to the thickness of an unsaturated aquifer, drawdowns are small enough to warrant the use of linearized flow equations,
- Drawdowns are small enough to provide permanent stream-aquifer hydraulic connection,
- Streambed cross section has horizontal and vertical dimensions that are small compared to the aquifer thickness,
- Stream is located along the y axis and is of infinite extent ($-\infty < y < \infty$),
- Seepage flow rates between stream and aquifer are proportional to the difference in piezometric head across the streambed,
- Alluvial aquifer is separated from a source bed with constant head with an incompressible aquitard of hydraulic conductivity k_a ($k_a \ll k$) and thickness m_a
- The hydrologic system (i.e., the aquifer, stream, and source bed) is in the state of equilibrium before the commencement of pumping.

Under these assumptions, the flow problem can be described by (Hantush, 1964; Hunt, 1999; Butler et al., 2001; and Zlotnik, 2004)

$$T\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right) = S\frac{\partial h}{\partial t} + w \tag{1a}$$

where

$$w = Q\delta(x - l)\delta(y) - \lambda(H - h)\delta(x) - \frac{k_a}{m_a}(H - h) - R, \tag{1b}$$

T is transmissivity, H is hydraulic head in the aquifer, stream and source bed at time $t = 0$, R is groundwater reacharge, and λ is the streambed characteristic. For streambeds with small horizontal and vertical dimensions, the latter can be approximated by $\lambda \approx k_s w_s/m_s$,

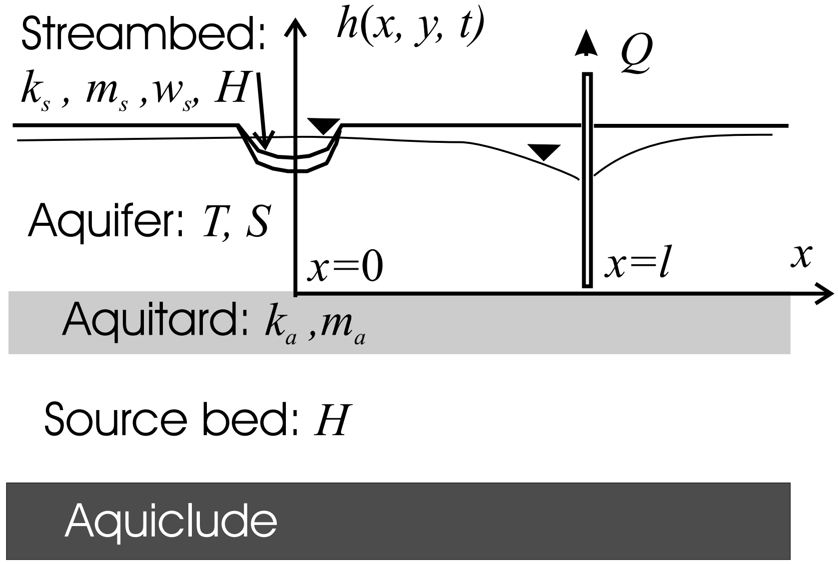


Figure 1: A schematic representation of a stream-aquifer-aquitard system and major hydrological parameters.

where k_s, w_s and m_s are the hydraulic conductivity, width and thickness of the streambed, respectively (Hunt et al., 2001).

Flow equation (1) is subject to the initial and boundary conditions

$$h(x, y, 0) = H, \qquad \lim_{x^2+y^2 \to \infty} h = H, \tag{2}$$

respectively.

The stream depletion rate q is defined by (e.g., Hunt, 1999, Eq. 6)

$$q = -\lim_{\epsilon \to 0} \int_{-\infty}^{\infty} \int_{-\epsilon}^{\epsilon} w dx dy = \lambda \int_{-\infty}^{\infty} [H - h(0, y, t)] dy. \tag{3}$$

In terms of drawdown $\phi(x, y, t) = H - h(x, y, t)$ and the dimensionless parameters

$$\phi_d = \frac{\phi T}{Q}, \qquad t_d = \frac{Tt}{Sl^2}, \qquad x_d = \frac{x}{l}, \qquad B_d^2 = \frac{m_a T}{k_a l^2}, \qquad \lambda_d = \frac{\lambda l}{T} \tag{4}$$

equations (1) – (3) can be rewritten as

$$\nabla_d^2 \phi_d = \frac{\partial \phi_d}{\partial t_d} - \delta(x_d - 1)\delta(y_d) + \lambda_d \phi_d \delta(x_d) + B_d^{-2} \phi_d \tag{5}$$

subject to the initial and boundary conditions

$$\phi_d(x_d, y_d, 0) = 0, \qquad \lim_{x_d^2+y_d^2 \to \infty} \phi_d = 0, \tag{6}$$

respectively. The stream depletion rate is given by

$$\frac{q}{Q} = \lambda_d \int_{-\infty}^{\infty} \phi_d(0, y_d, t_d) dy_d. \tag{7}$$

Note that, analogous to Jenkins (1968), dimensionless time t_d is scaled with the stream depletion factor Sl^2/T. Dimensionless parameter B_d accounts for the effects of leakage, such that $B_d = \infty$ corresponds to a non-leaky aquifer and $B_d = 0$ to a perfect connection with a source bed. Dimensionless parameter λ_d accounts for water exchange between the stream and the aquifer, such that $\lambda_d = 0$ indicates the absence of a stream and $\lambda_d = \infty$ corresponds to a perfect stream-aquifer connection, i.e., to the full aquifer penetration by a stream. Dimensional counterparts of B_d and λ_d were used by Hantush (1964) and Hunt (1999), respectively.

Drawdown and Stream Depletion Rate

Let $\Phi(x_d, \alpha, p)$ be the Laplace-Fourier transform of ϕ_d defined as

$$\bar{\phi}(x_d, y_d, p) = \int_0^\infty \phi(x_d, y_d, t_d)\mathrm{e}^{-pt_d} dt_d, \qquad \Phi(x_d, \alpha, p) = \int_{-\infty}^\infty \bar{\phi}(x_d, y_d, p)\mathrm{e}^{i\alpha y_d} dy_d. \tag{8}$$

Taking the Laplace-Fourier transform of (5) – (6) yields an ordinary differential equation

$$\frac{d^2\Phi}{dx_d^2} - \beta^2\Phi = -\frac{1}{p}\delta(x_d - 1) + \lambda_d \Phi \delta(x_d), \qquad \beta^2 = \alpha^2 + p + B_d^{-2} \tag{9}$$

subject to the boundary condition

$$\lim_{x_d \to \pm\infty} \Phi = 0. \tag{10}$$

Following Hunt (1999), the solution of (9) – (10) can be written as

$$\Phi = \Phi_1 - \Phi_2, \qquad -\infty < x_d < \infty, \tag{11}$$

where

$$\Phi_1 = \frac{1}{2\beta p}\mathrm{e}^{-\beta|x_d - 1|} \tag{12}$$

and

$$\Phi_2 = \frac{\lambda_d}{2\beta p(2\beta + \lambda_d)}\mathrm{e}^{-\beta(1+|x_d|)}. \tag{13}$$

Following Hunt (1999, Eq. 25), we note that in the absence of a stream ($\lambda = \lambda_d = 0$), our solution must reduce to a well known solution by Hantush and Jacob (1955). Hence Φ_1 in (12) is a Laplace-Fourier image of the Hantush-Jacob solution for an observation well located at dimensionless distance $r_d^2 = (x_d - 1)^2 + y_d^2$,

$$\phi_{d1}(x_d, y_d, t_d) = \frac{1}{4\pi} W\left(u, \frac{r_d}{B_d}\right), \tag{14}$$

where $u = r_d^2/(4t_d)$ and W is a well function defined by Hantush (1964) as

$$W(u, x) = \int_u^\infty \frac{1}{y} \exp\left(-y - \frac{x^2}{4y}\right) dy. \tag{15a}$$

The relevant properties of the well function W are

$$W(u, \infty) = 0, \qquad W(u, 0) = E_1(u), \qquad W(0, x) = 2K_0(x), \tag{15b}$$

where E_1 is the exponential integral, and K_0 is the modified Bessel function of the second kind.

The Laplace-Fourier transform of (13) is inverted using Hunt's (1999, Eq. 29) technique,

$$\phi_{d2}(x_d, y_d, t_d) = \frac{1}{4\pi} \int_0^\infty \mathrm{e}^{-\theta} W\left(u_\lambda, \frac{r_\lambda}{B_d}\right) d\theta. \tag{16}$$

where $r_\lambda^2 = (1 + |x_d| + 2\theta/\lambda_d)^2 + y_d^2$ and $u_\lambda = r_\lambda^2/(4t_d)$. Hence the dimensionless drawdown $\phi_d = \phi_{d1} - \phi_{d2}$, or its dimensional counterpart $\phi = \phi_1 - \phi_2$, is given by

$$\phi(x, y, t) = \frac{Q}{4\pi T} W\left(u, \frac{r_d}{B_d}\right) - \frac{Q}{4\pi T} \int_0^\infty \mathrm{e}^{-\theta} W\left(u_\lambda, \frac{r_\lambda}{B_d}\right) d\theta. \tag{17}$$

The Laplace image of the stream depletion rate in (7) can be obtained from the Laplace-Fourier transform of the drawdown in (8) as

$$\frac{\overline{q}}{Q} = \lambda_d \int_{-\infty}^\infty \overline{\phi}(0, y_d, t_d) dy_d = \lambda_d \Phi(0, 0, p). \tag{18}$$

Substituting (11) into (18) gives a Laplace image of the stream depletion rate

$$\frac{\overline{q}}{Q} = \frac{\lambda_d}{p(2\beta_0 + \lambda_d)} \mathrm{e}^{-\beta_0}, \qquad \beta_0^2 = p + B_d^{-2}. \tag{19}$$

Using an expansion

$$\frac{2\lambda}{(p - 1/B^2)(2\sqrt{p} + \lambda_d))} = \frac{Ba_1}{\sqrt{p} - 1/B} + \frac{Ba_2}{\sqrt{p} + 1/B} - \frac{4}{\lambda} \frac{a_1 a_2}{\sqrt{p} + \lambda_d/2} \tag{20}$$

in (19), where

$$a_1 = \frac{B_d}{2/\lambda_d + B_d}, \qquad a_2 = \frac{B_d}{2/\lambda_d - B_d}, \tag{21}$$

and taking the inverse Laplace transform of each term separately (Carslaw and Jaeger, 1959; Appendix V, Eq. 12) gives

$$\begin{aligned} \frac{q}{Q} =& \frac{a_1}{2} \mathrm{e}^{-1/B_d} \mathrm{erfc}\left(\frac{1}{2\sqrt{t_d}} - \frac{\sqrt{t_d}}{B_d}\right) - \frac{a_2}{2} \mathrm{e}^{1/B_d} \mathrm{erfc}\left(\frac{1}{2\sqrt{t_d}} + \frac{\sqrt{t_d}}{B_d}\right) \\ &+ a_1 a_2 \mathrm{e}^{\lambda_d/2 + \lambda_d^2 t_d/4 - t_d/B_d^2} \mathrm{erfc}\left(\frac{1}{2\sqrt{t_d}} + \frac{\lambda_d \sqrt{t_d}}{2}\right). \end{aligned} \tag{22}$$

Our general solutions for drawdown (17) and stream depletion rate (22) contain a number of new and previously known solutions as special cases. These are discussed below.

Analysis of drawdown

The steady-state drawdown $\phi_{\rm st}$ is obtained from (17) by taking the limit as $t_d \to \infty$ and recalling (15b),

$$\phi_{\rm st}(x,y) = \frac{Q}{2\pi T} K_0\left(\frac{r_d}{B_d}\right) - \frac{Q}{2\pi T}\int_0^\infty {\rm e}^{-\theta} K_0\left(\frac{r_\lambda}{B_d}\right) d\theta. \tag{23}$$

The Theis solution for flow to a pumping well in a non-leaky aquifer without a stream is obtained from (17) by taking limit as $B_d \to \infty$ and $\lambda_d \to 0$.

The Theis solution for flow to a pumping well in a non-leaky aquifer with a perfect stream-aquifer connection is obtained from (17) by taking limit as $B_d \to \infty$ and $\lambda_d \to \infty$.

The Hantush-Jacob solution for flow to a pumping well in a leaky aquifer without a stream is obtained from (17) by taking the limit as $\lambda_d \to 0$ (or $r_\lambda \to \infty$) and recalling (15b),

$$\phi^{HJ}(x,y,t) = \frac{Q}{4\pi T} W\left(u, \frac{r_d}{B_d}\right). \tag{24}$$

The modified Hantush solution for flow to a pumping well near a fully penetrating stream is obtained from (17) by taking the limit as $\lambda_d \to \infty$. Since

$$\lim_{\lambda_d\to\infty} \phi_{d2}(x_d,y_d,t_d) = \frac{1}{4\pi} W\left(u_M, \frac{r_{Md}}{B_d}\right), \qquad r_{Md}^2 = (1+|x_d|)^2 + y_d^2, \text{and } u_M = \frac{r_{Md}^2}{4t_d}, \tag{25}$$

where r_{Md} is dimensionless distance between an observation point (x_d, y_d) and a mirrow image of the pumping well with respect to the stream, the modified Hantush solution is given by

$$\phi^{MH}(x,y,t) = \frac{Q}{4\pi T} W\left(u, \frac{r_d}{B_d}\right) - \frac{Q}{4\pi T} W\left(u_M, \frac{r_{Md}}{B_d}\right). \tag{26}$$

The transient Hunt (1999) *solution* for flow to a pumping well in a single-unit aquifer near a shallow partially penetrating stream is obtained from (17) by taking the limit as $B_d \to \infty$ and recalling (15b),

$$\phi^{Hu}(x,y,t) = \frac{Q}{4\pi T} E_1(u) - \frac{Q}{4\pi T}\int_0^\infty {\rm e}^{-\theta} E_1(u_\lambda)\, d\theta. \tag{27}$$

which is identical to equations (25) and (29) of Hunt (1999). The steady-state counterpart of (27) exists for $\lambda_d > 0$ and is given by equation (5) of Kollet et al. (2002),

$$\phi_{\rm st}^{Hu} = \frac{Q}{4\pi T}\ln\left[\frac{(1+|x_d|)^2 + y_d^2}{(1-x_d)^2 + y_d^2}\right] + \frac{Q}{2\pi T}\int_{1+|x_d|}^\infty \frac{\eta}{\eta^2 + y_d^2} {\rm e}^{-\lambda_d \frac{\eta - 1 - |x_d|}{2}} d\eta. \tag{28}$$

Figure 2 illustrates the effect of leakage ($B_d = 10$ and $B_d = 100$) on the cone of depression. Leakage (the smaller B_d, the larger the leackage) reduces the lateral extent of the cone of depression. For example, the equipotential $\phi_d = 0.3$ barely reaches the stream for a larger leakage $B_d = 10$. This effect can be explained be the presence of a source bed.

Figure 3 compares the time behavior of the normalized local drawdown ϕ_d in the observation well located at $(x_d, y_d) = (0.2, 0.0)$ for $B_d = 10$ and $B_d = 100$. The latter case corresponds to an example presented by Hunt's (1999) Figure 6. For smaller B_d (i.e., $B_d = 10$), drawdown practically reaches the steady state at earlier times, because of the the increased leakage across the aquitard to the pumped aquifer at earlier times.

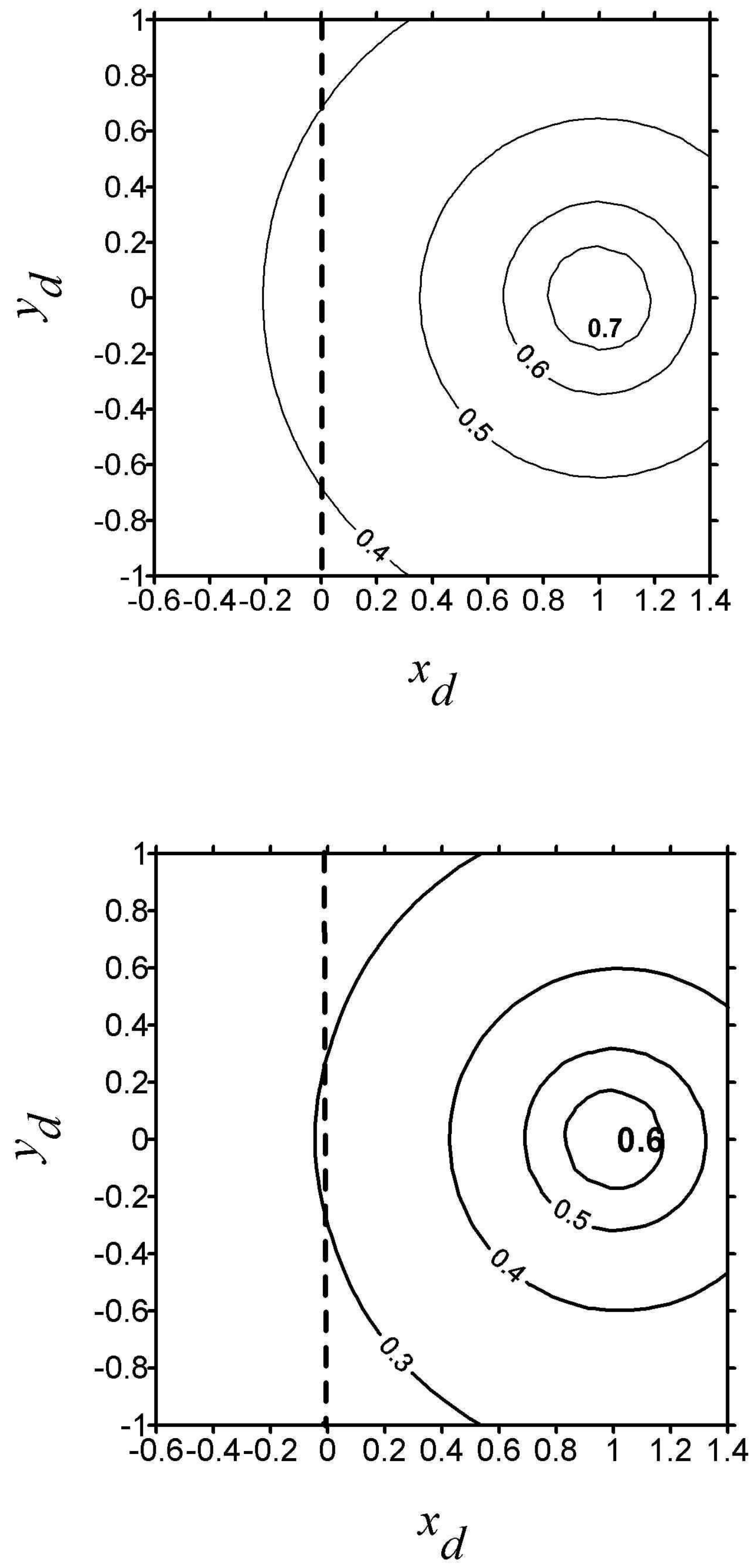
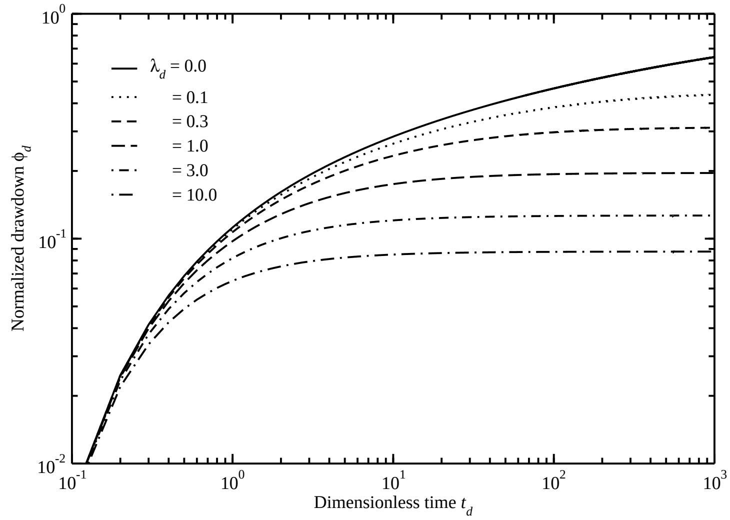
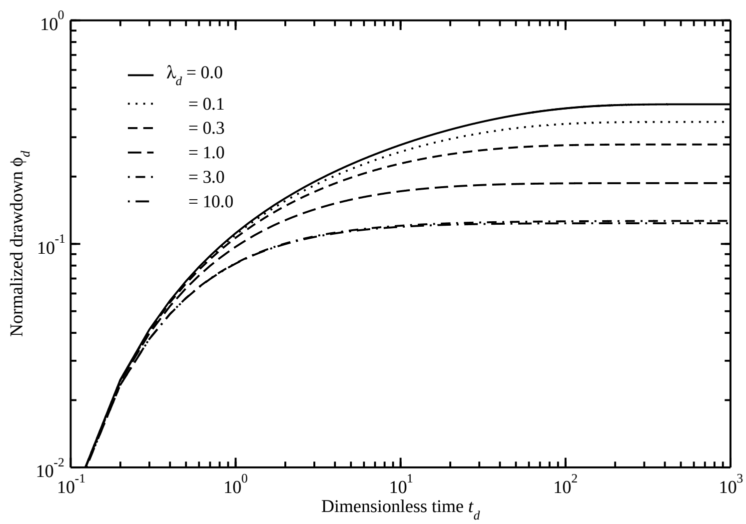


Figure 2: Normalized drawdown $\phi_d = \phi T/Q$ at dimensionless time $t_d = 100$ for $\lambda_d = 0.1$ and (a) $B_d = 100$ and (b) $B_d = 10$. The stream is located at $x_d = 0$ and is indicated by the dashed line.



(a)



(b)

Figure 3: Normalized drawdown $\phi_d = \phi T/Q$ at $(x_d, y_d) = (0.2, 0.0)$ as a function of dimensionless time $t_d = Tt/(Sl^2)$ for two values of the dimensionless leakage parameter, (a) $B_d = 100$ and (b) $B_d = 10$, and several values of the dimensionless streambed characteristics λ_d.

Analysis of stream depletion rate (SDR)

General expression (22) for SDR accounts for both streambed-aquifer water exchange and leakage across the aquitard. This solution corresponds to the dimensionless streambed conductance $\lambda_d \geq 0$ and leakage $B_d \leq \infty$, and contains several important special cases.

Maximal Stream Depletion Rate (MSDR), defined by Zlotnik (2004) as a maximum fraction of the pumping rate supplied by SDR, corresponds to *the steady-state* SDR and is obtained from (22) by taking the limit as $t_d \to \infty$,

$$\frac{q_{\mathrm{st}}}{Q} = a_1 \mathrm{e}^{-1/B_d} = \frac{\lambda}{2\sqrt{k_a T/m_a} + \lambda} \exp\left(-l\sqrt{\frac{k_a}{m_a T}}\right). \tag{29}$$

This equation demonstrates that both leakage from the source bed and the stream contribute to the MSDR. The leakage causes the MSDR to decrease exponentially with distance from the well to the stream, while the effect of the stream-aquifer connection (determined by parameter λ) is independent of this distance. It is important to note that MSDR < 1.

The Hantush (1955, 1964) *solution* for the SDR induced by a well pumping in a leaky aquifer near a fully penetrating stream is obtained by taking the limit of (22) as $\lambda_d \to \infty$,

$$\frac{q^H}{Q} = \frac{1}{2}\mathrm{e}^{-1/B_d}\mathrm{erfc}\left(\frac{1}{2\sqrt{t_d}} - \frac{\sqrt{t_d}}{B_d}\right) + \frac{1}{2}\mathrm{e}^{1/B_d}\mathrm{erfc}\left(\frac{1}{2\sqrt{t_d}} + \frac{\sqrt{t_d}}{B_d}\right). \tag{30}$$

The MSDR is obtained from (30) by taking the limit as $t_d \to \infty$,

$$\mathrm{MSDR}^H = \mathrm{e}^{-1/B_d} = \exp\left(-l\sqrt{\frac{k_a}{m_a T}}\right). \tag{31}$$

The exponential term indicates again the strong attenuation of the MSDR with increase of distance between the well and the stream. Other factors that may affect MSDR are alluvial valley width and availability of other sources of the aquifer recharge (Zlotnik, 2004).

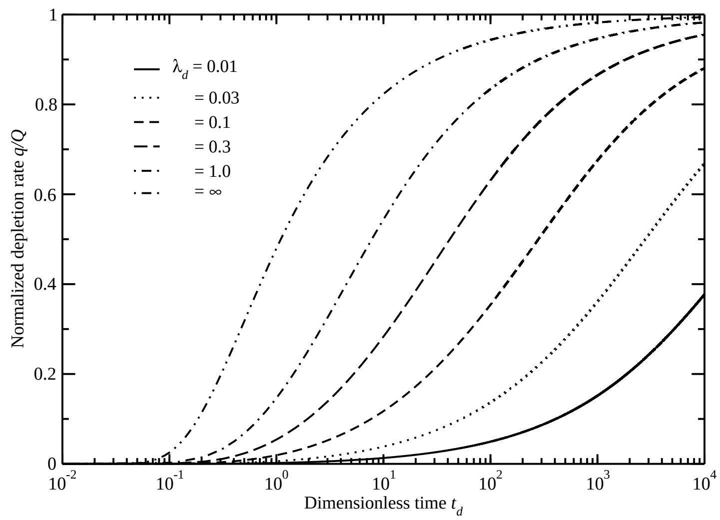
The Theis-Glover-Balmer solution for the SDR induced by a well pumping in a non-leaky aquifer near a fully penetrating stream is obtained from (22) by taking the limit as both $B_d \to \infty$ and $\lambda_d \to \infty$,

$$\frac{q^{TGB}}{Q} = \mathrm{erfc}\left(\frac{1}{2\sqrt{t_d}}\right). \tag{32}$$

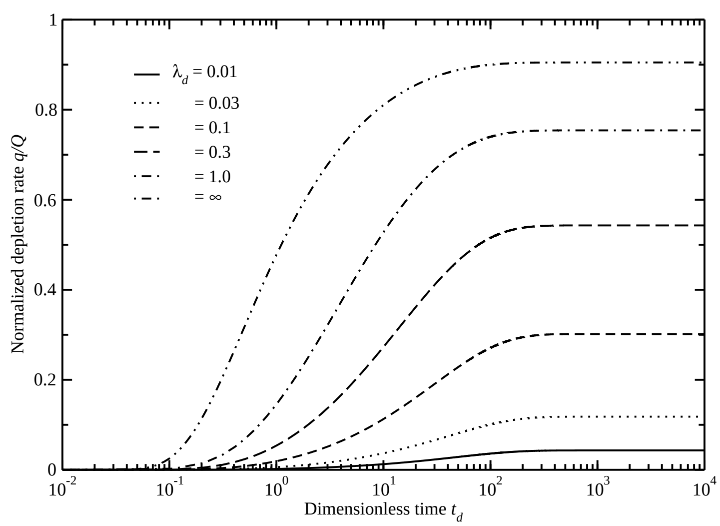
The comparison of (30) and (32) reveals that the assumption of full penetration overestimates the magnitude of the MSDR.

The Hunt (1999) *transient solution* for the SDR induced by a pumping well in a non-leaky aquifer near a shallow partially penetrating stream is obtained by taking the limit of (22) as $B_d \to \infty$, i.e., by assuming an impermeable aquitard,

$$\frac{Q^{Hu}}{Q} = \mathrm{erfc}\left(\frac{1}{2\sqrt{t_d}}\right) - \mathrm{e}^{\lambda_d/2+\lambda_d^2 t_d/4}\mathrm{erfc}\left(\frac{1}{2\sqrt{t_d}} + \frac{\lambda_d\sqrt{t_d}}{2}\right). \tag{33}$$



(a)



(b)

Figure 4: Normalized stream depletion rate q/Q as a function of dimensionless time $t_d = Tt/(Sl^2)$ for two values of the dimensionless leakage parameter, (a) $B_d = \infty$ and (b) $B_d = 10$, and several values of the dimensionless streambed characteristics λ_d.

Both the Theis-Glover-Balmer and Hunt solutions for the SDR indicate that in non-leaky aquifers the pumping rate is fully supplied by the stream depletion after extended pumping time. Hence, the MSDR eventually reaches 1 (Zlotnik, 2004).

Figure 4 illustrates the combined influence of partial stream penetration and aquifer leakage on both the SDR and MSDR. First, a family of Hunt (1999) curves for the SDR in a non-leaky aquifer with a partially penetrating stream is plotted in Figure 4a as a function of dimensionless time t_d. It is given by (33) and includes the Theis-Glover-Balmer solution for a case of perfect stream-aquifer connection ($\lambda_d = \infty$). While Hunt's SDR (33) is obtained from (22) by taking the limit as $B_d \to \infty$, it is within 9% of the SDR given by (22) at $B_d = 100$, and practically attains this limit at $B_d = 500$. By the same token, the Theis-Glover-Balmer SDR (32) is obtained from (22) with $B_d \to \infty$ (or $B_d = 500$) and $\lambda_d \to \infty$ ($\lambda_d = 10$). Needless to say, MSRD = 1 for all these curves.

Figure 4b shows a family of curves for the SDR in a leaky aquifer with $B_d = 10$. The comparison of Figures 4a and 4b reveals that the larger B_d, the longer it takes for the stream depletion to reach steady state. Leakage across the aquitard causes the MSDR to decay exponentially with the distance between the well and the stream and prevents it from reaching 1. The leakage also leads to an earlier stabilization of the MSDR after the commensment of pumping. The time it takes for each SDR curve to reach the corresponding MSDR may differ, and sensitivity analysis to the stream-aquifer-aquitard parameters would be appropriate (Christensen, 2001).

Conclusions

We obtained transient solutions for drawdown and stream depletion rate (SDR), which allow one to elucidate the combined effect of streambed leakage, stream penetration, and aquifer leakage. To analyze the SDR, we used a concept of the Maximal Stream Depletion Rate (MSDR), which is defined as a maximum fraction of the pumping rate supplied by stream depletion. Stream depletion rate reaches the MSDR after the hydrologic system arrives at a new equilibrium after the start of pumping. Stream depletion may only partially support groundwater withdrawal from a pumping well in leaky aquifers. The balance of groundwater withdrawals that is not supported by the stream depletion can be supplied from other sources.

In general, the MSDR can be assessed only with full consideration of hydrogeological conditions that include the hydrostratigraphy, hydraulic properties of the aquifer and streambed, geometry of recharge and discharge zones, and location of pumping well. The obtained solutions may be used for assessment of MSDR and will complement numerical techniques that are applied for detailed evaluation of stream-aquifer water budgets.

For this purpose, parameters of the aquifer, well, streambed, and aquitard must be available a priori. If this is not a case, solutions can be used for designing the aquifer testing programs similar to these by Sophocleous et al. (1988), Hunt et al. (2001), Nyholm et al. (2003), and Kollet and Zlotnik (2003). The pre-testing designs can be significantly enhanced by a sensitivity analysis of the on-site conditions (Christensen, 2000).

References

Butler, J.J., Jr., V.A. Zlotnik, M.-S. Tsou, 2001, Drawdown and stream depletion produced by pumping in the vicinity of a partially penetrating stream, Ground Water, 39(5), 651-659.

Carslaw, H. S. and J. C. Jaeger, 1959, *Conduction of heat in solids*, 2nd ed., Oxford Univ. Press, New York.

Christensen, S., 2000, On the estimation of stream flow depletion parameters by drawdown analyses, Ground Water, 38(5), 726-734.

Glover, R.E., and G.G. Balmer, 1954, River depletion resulting from pumping a well near a river, Trans. Am. Geophys. Union, v. 35, 468-470.

Hantush, M. S., 1955, Discussion of "River depletion resulting from pumping a well near a river," by Robert E. Glover and Glenn G. Balmer," Trans. Am. Geophys. Union, 36(2), 345-346.

Hantush, M.S., 1964, Hydraulics of Wells, in Advances in Hydroscience, ed. V. Chow, Academic Press, New York, v.1, 281-432.

Hantush, M.S., 1965, Wells near streams with semipervious beds, J. Geophysical Res., 70(12), 2829-2838.

Hantush, M.S. and C.E. Jacob, 1955, Non-steady radial flow in an infinite leaky aquifers, Trans. Am. Geophys. Union, 36, 95-112.

Hunt, B., 1999, Unsteady stream depletion from ground-water pumping, Ground Water, 37(1), 98-102.

Hunt, B., J. Weir, and B. Clausen, 2001, A stream depletion field experiment, Ground Water, 39(2), 283-289.

Jenkins, C. T., 1968. Techniques for computing rate and volume of stream depletion by wells, Ground Water, 6(2), 37-46.

Kollet, S. and V.A. Zlotnik, 2003, Stream depletion predictions using data of pumping tests in heterogeneous stream-aquifer system in the Great Plains, USA, J. Hydrology, v. 281(1-2), 96-114.

Nyholm, T., Christensen, S., Rasmussen, K.R., 2002. Flow depletion in a small stream caused by ground water abstraction from wells. Ground Water, 40(4), 425-437.

Nyholm, T., K.R. Rasmussen, S. Christensen, 2003, Estimation of stream depletion and uncertainty from discharge measurements in a small alluvial stream, J. Hydrology, v. 274, 129-144.

Sophocleous, M., M.A. Townsend, L.D. Vogler, T.J. McLain, E.T. Marks, and G.R. Goble, 1988, Experimental studies in stream-aquifer interactions along the Arkansas River in Central Kansas - field testing and analysis, J. Hydrology, 98, 249-273.

Theis, C.V., 1941, The effect of a well on the flow of a nearby stream, EOS Transactions, AGU, v. 22, 734-738.

Zlotnik, V.A., 2004, A concept of maximum stream depletion rate for leaky aquifers in alluvial valleys, Water Resour. Res., 40, W06507, doi:10.1029/2003WR002932.

Zlotnik, V.A., H. Huang, and J.J. Butler, 1999, Evaluation of stream depletion considering finite stream width, shallow penetration, and proerties of streambed sediments, in *Proceedings of Water 99, Joint Congress*, pp. 221-226, Inst. of Eng., Barton ACT, Australia.