Suspensions of microswimmers at small and moderate concentrations: effective shear viscosity and dynamics

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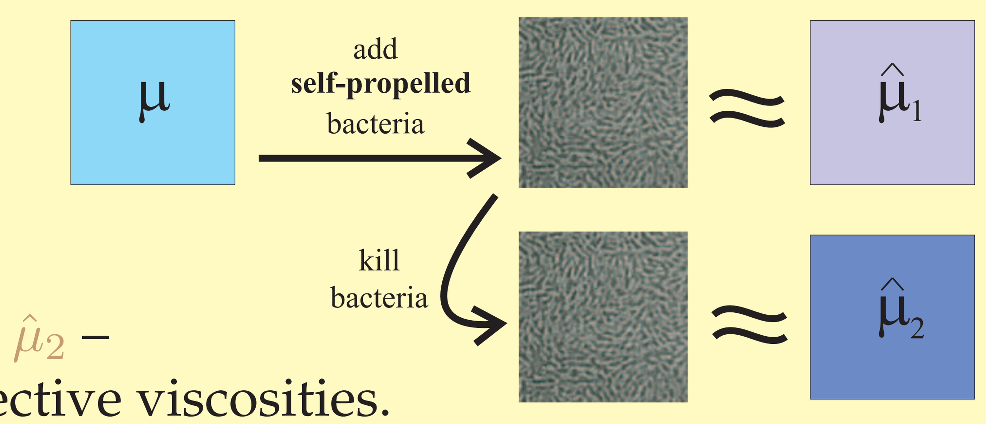
Motivating experiment



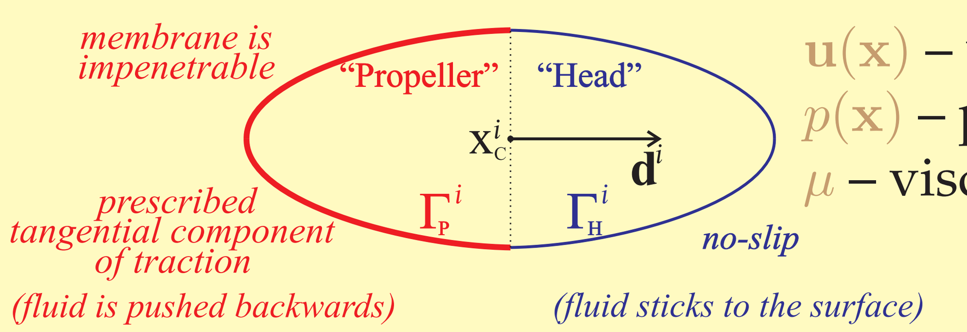
$\hat{\mu}_1$, $\hat{\mu}_2$ – effective viscosities.

I. Aronson, A. Sokolov (experiments): $\hat{\mu}_1 \ll \hat{\mu}_2$.

$\hat{\mu}_1$ can be 5-7 times smaller than $\hat{\mu}_2$ for moderate concentrations. Possible: $\hat{\mu}_1 < \mu$.

Sharp contrast with passive inclusions: rigid inclusions always increase effective viscosity.

Model (well-posed)



$\mathbf{u}(\mathbf{x})$ – velocity; $p(\mathbf{x})$ – pressure; μ – viscosity.

$$\begin{cases} \mu\Delta\mathbf{u} = \nabla p \\ \operatorname{div}(\mathbf{u}) = 0 \end{cases} \quad \text{in } \Omega_F = \Omega \setminus B.$$

Rigid swimmer: $\mathbf{v}(\mathbf{x}) = \mathbf{v}_C + (\mathbf{x} - \mathbf{x}_C) \times \omega$,

On "forward" (head) part: Fluid sticks to the swimmer

$$\mathbf{u}(\mathbf{x}) = \mathbf{v}(\mathbf{x}) \quad x \in \Gamma_H, \quad \text{no-slip.}$$

On "back" (propulsion) part:

$$\begin{cases} (\mathbf{u}(\mathbf{x}) - \mathbf{v}(\mathbf{x})) \cdot \mathbf{n} = 0 & \text{no penetration, slip is allowed,} \\ \tau\sigma(\mathbf{u}, p)\mathbf{n} - \text{given} & \text{partially prescribed traction.} \end{cases}$$

τ – unit tangent to the surface, $\tau \cdot \mathbf{d} \le 0$.

$f_p := \int_{\Gamma_P} \tau\sigma(\mathbf{u}, p)\mathbf{n}\, dx$ – propulsion strength of swimmer.

Balance conditions for the whole swimmer:

$$\int_{\Gamma_H \cup \Gamma_P} \sigma(\mathbf{u}, p)\mathbf{n}\, dx = 0 \quad \text{Balance of forces}$$

$$\int_{\Gamma_H \cup \Gamma_P} (\mathbf{x} - \mathbf{x}_C) \times \sigma(\mathbf{u}, p)\mathbf{n}\, dx = 0 \quad \text{Balance of torque.}$$

Dynamics of the swimmer: $\dot{\mathbf{x}}_C = \mathbf{v}_C \quad \dot{\mathbf{d}}^i = \mathbf{d}^i \times \omega$.

Measuring shear viscosity

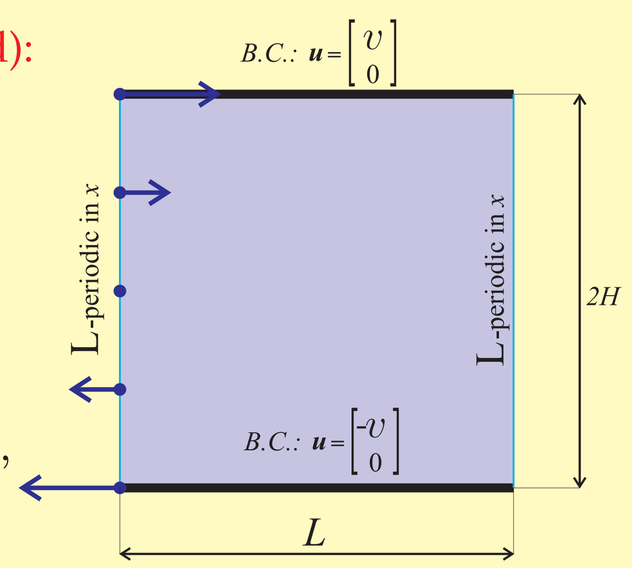
Shear viscosity (homog. fluid): $\bar{\mu} := \frac{H}{L}\frac{(\mathbf{F}_T - \mathbf{F}_B)\cdot \mathbf{e}_1}{2v} = \mu$.

Inhom. fluid (suspension) ⇒ $\mathbf{F}_T(t), \mathbf{F}_B(t) \Rightarrow \bar{\mu} = \bar{\mu}(t)$ – instant. apparent viscosity (material & state property).

Effective viscosity:

$$\hat{\mu} := \lim_{T\to\infty} \frac{1}{T}\int_0^T \bar{\mu}(t)\, dt,$$

(material property).



Small concentrations (no swimmer-swimmer interactions)

Dilute assumptions:

(i) swimmers interact only with the background flow (swimmer-swimmer interactions can be ignored);

(ii) only orientations (not positions) of swimmers play role in the effective viscosity;

Dilute assumptions ⇒ analyze one swimmer.
Many swimmers = sum of effects due to individual swimmers.

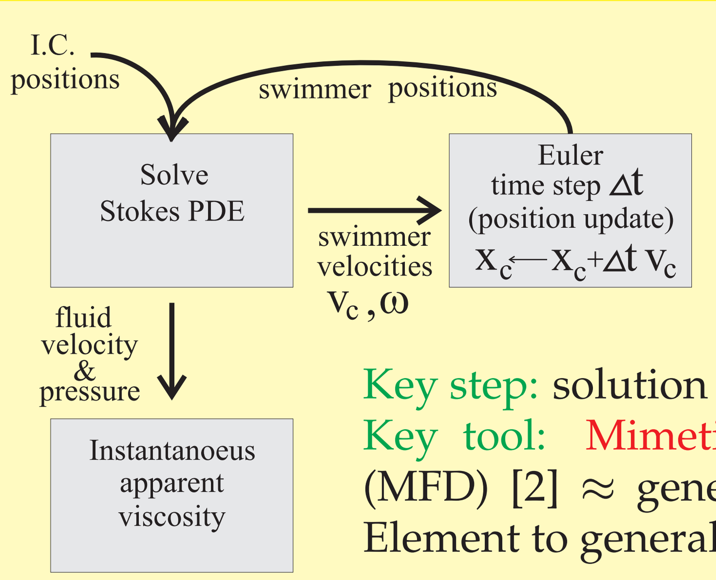
THM: Dilute assumption ⇒ $\hat{\mu}(f_p) = \hat{\mu}(0)$, no dependance on f_p.

Remark: Adding rotational noise to the model breaks symmetry in $p(\theta)$. Leads to preferential alignment of swimmers (Leal & Hinch): $\hat{\mu}(f_p) < \hat{\mu}(0)$ for $f_p > 0$ (Haines, Karpeev, Aronson, Berlyand).

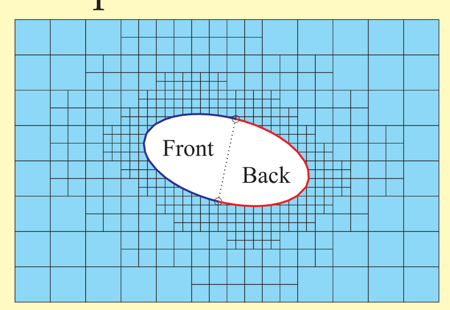
Key steps:

1. Rotational velocity of swimmer $\omega(\theta, f_p) = \omega(\theta)$ is even: $\omega(\theta) = \omega(-\theta)$.
2. Density function $p(\theta)$, time spent around angle θ, is even: $p(\theta) = p(-\theta)$.
3. Contribution $\bar{\eta}(\theta, f_p) := \bar{\mu}(\theta, f_p) - \bar{\mu}(\theta, 0)$ to instantaneous apparent viscosity $\bar{\mu}(\theta, f_p)$ due to self propulsion is odd: $\bar{\eta}(-\theta, f_p) = -\bar{\eta}(\theta, f_p)$.
4. Overall contribution to effective viscosity from self-propulsion:
$$\hat{\eta}(f_p) := \hat{\mu}(f_p) - \hat{\mu}(0) = \int_{-\pi}^{\pi} p(\theta)\bar{\eta}(\theta)\, d\theta = 0.$$

Moderate concentrations: numerical solution scheme (all interactions)



Sample mesh:



Key step: solution of Stokes PDE.
Key tool: Mimetic Finite Difference (MFD) [2] ≈ generalization of Finite Element to general polygonal meshes.

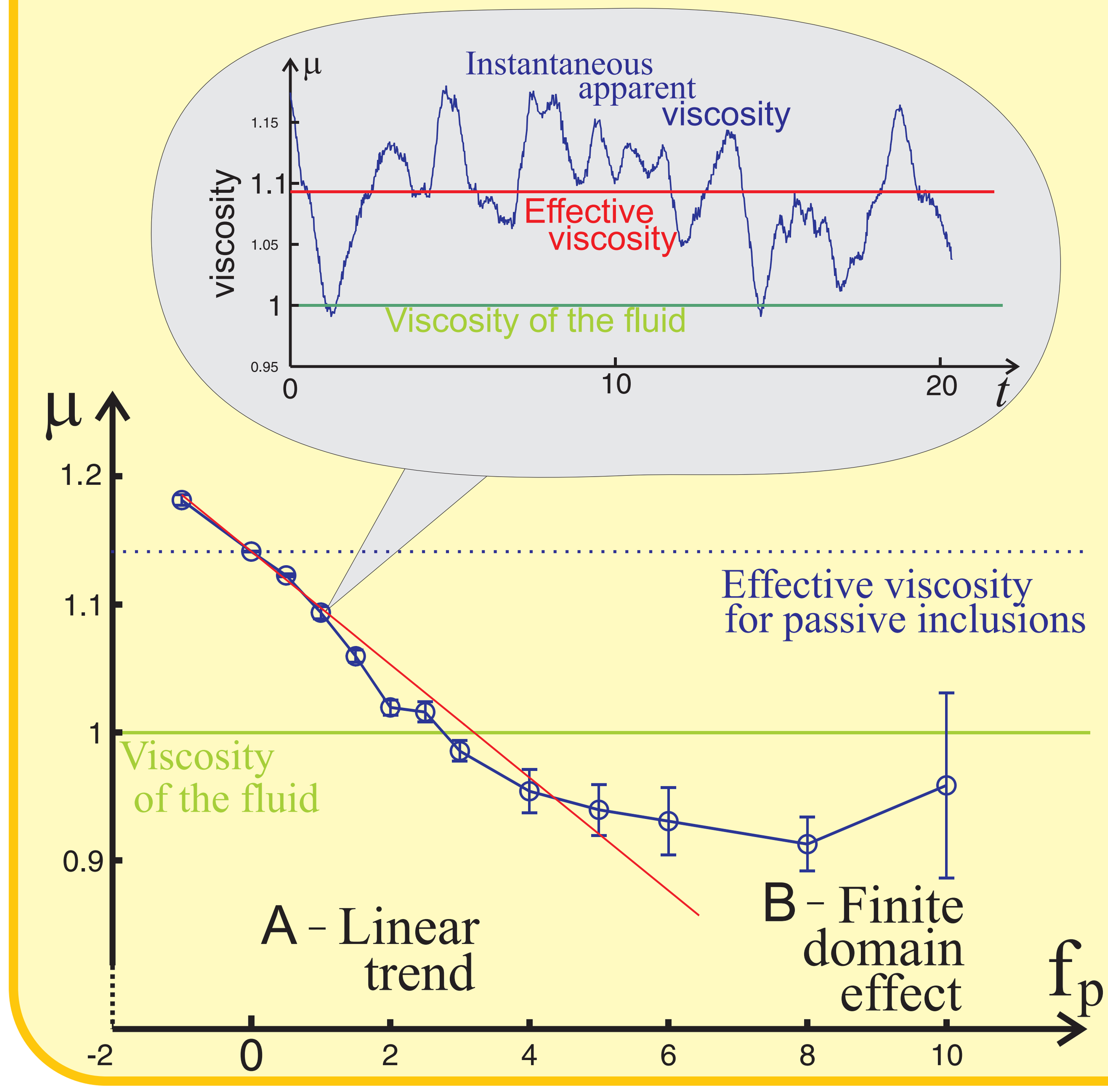
Advantages of MFD: Performance & flexibility of use and extension to time dependent Stokes, Navier-Stokes.

Some computational issues:

- **optimal time step** $\triangle t$:
  - too large $\triangle t$ → inaccurate dynamics → inaccurate measurement to effective viscosity.
  - too small $\triangle t$ → too short observation time → inaccurate measurement to effective viscosity.
- **collisions of swimmers** (due to finite $\triangle t$).

Moderate concentrations: results

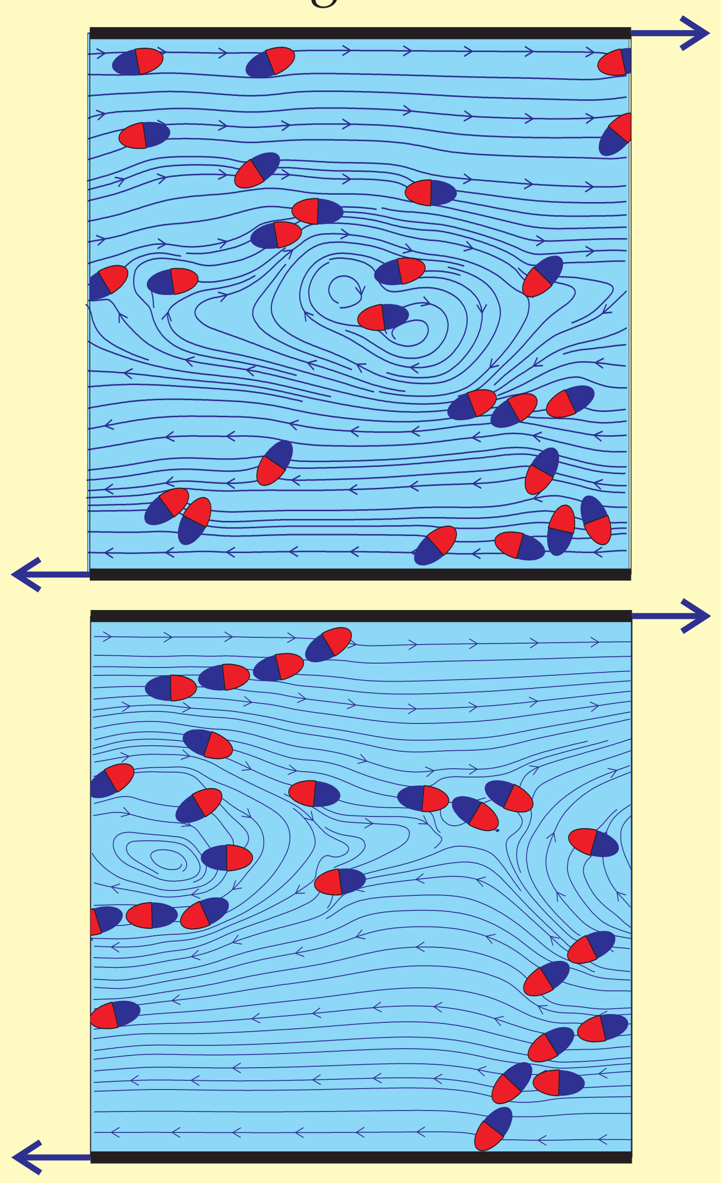
Effective viscosity $\hat{\mu}(f_p)$ as a function of f_p:



Tendency for alignment (pattern formation) is observed even in the presence of background flow:

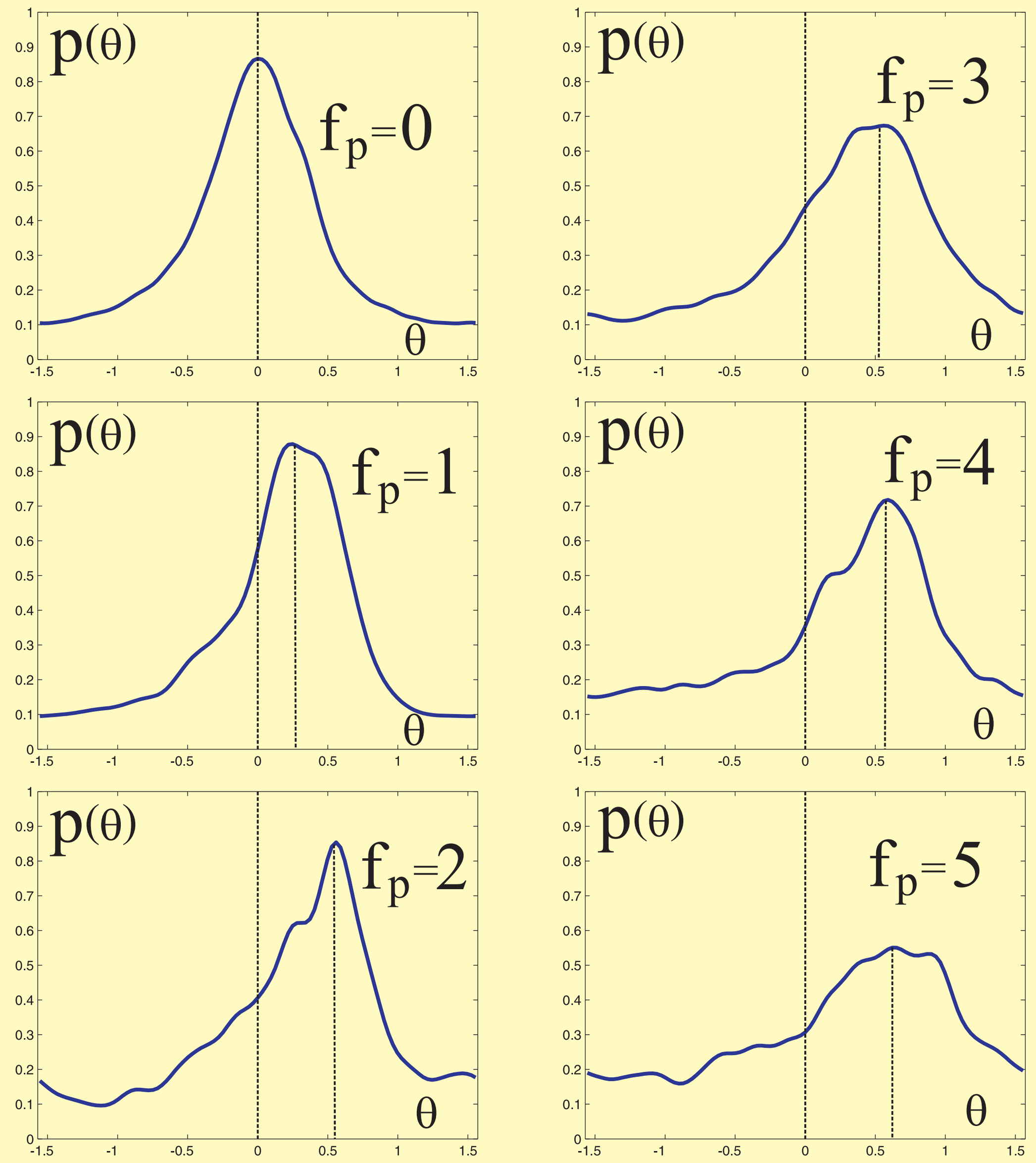
Pushers ($f_p > 0$, effective propulsion force behind center) tend to swim side-by-side.

Pullers ($f_p < 0$, effective propulsion force in front of center) tend to swim head-to-tail forming train-like structures.

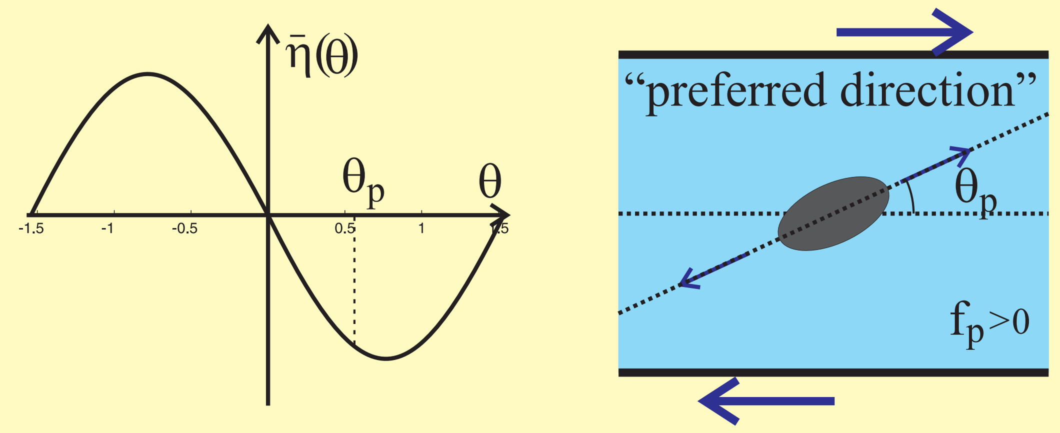


Why reduction of viscosity?

Hydrodynamic interactions ∼ rotational noise: break up of the symmetry in $p(\theta)$ – peak in the density $p(\theta)$ shifts (preferential alignment).



Preferred direction – swimmer creates flow that aids ($f_p > 0$) the background shear flow → reduction of viscosity.



References

[1] *Effective shear viscosity and dynamics of suspensions of microswimmers at small and moderate concentrations*, **V. Gyrya**, K. Lipnikov, I. Aronson, L. Berlyand, submitted (2009).

[2] *Mimetic finite difference method for the Stokes problem on polygonal meshes*, L. Beirao da Veiga, **V. Gyrya**, K. Lipnikov, G. Manzini, *JCP*, vol. 228, no. 19, pp. 7215-7232 (2009).

[3] *High-order mimetic finite difference method for diffusion problems on polygonal meshes*, **V. Gyrya**, K. Lipnikov, *JCP*, vol. 227, no. 20, pp. 8841-8854 (2008).

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