2D Conservative Interpolation for ALE Methods

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During this summer I collaborated with Mikhail Shashkov and Burton Wendroff. Our work is related to the numerical solution of the fluid dynamics problems.

When solving hyperbolic PDEs describing some problem of fluid dynamics, we can use the Lagrangian approach - the mesh moves with the fluid. It may happen, that the grid becomes distorted and needs modification. Technique, when we modify the mesh and recompute the values on it to start a new time step, is called ALE (Arbitrary Lagrangian-Eulerian) method. When doing this, we need a general, accurate and efficient algorithm for conservative function interpolation (remapping). We want this interpolation to be accurate, conservative and monotonicity preserving. To achieve high-order of accuracy of interpolation, we use the piecewise linear reconstruction of functions on Lagrangian grid and then integrate over meshes of the modified grid. In 1D the situation is easy, all methods are fast enough on todays computers and we can choose from methods with the best properties.

In 2D the situation is much more complex and we have much more work to do with developing such an algorithm. The original method for computation of mean values on the new grid based by exact integration is very slow, we try to develop much faster method and keep the second-order of accuracy.

Our method based on swept area integration (swept area is the region created by moving one cell edge from the original grid to the edge of the new grid) is much faster and preserves the second order of accuracy and conservativity. This method has also its disadvantages. One of them is the possibility, that new values are out of global or local boundaries. We have developed several methods for repairing resulting means and have shown, that their applications do not decrease the order of accuracy in most cases.

We combined these methods for computation of means with several well known methods for computation of slopes. We have also improved one of the methods to preserve monotonicity of reconstructed function. We showed advantages and also disadvantages of such modification.

We have implemented all the methods. Using their combinations for means and slopes computation we solved a set of testing problems on different types of moving grids. Now errors of all these methods are easily comparable and one can choose the best method for his type of problem. We have also described all the codes in a manual, theoretical description of used methods and numerical results are included into the report, which will be available very soon.

This work followed [1] and the computed problems are the same to be comparable. Our results agree very well with the paper and also our new methods give us reasonable results.

References

[1] Margolin L. G., Shashkov M.: *Second-Order Sign-Preserving Remapping on General Grids*, LA-UR-02-525, LANL report, 2002.