Reference-Jacobian Smoother with Unstructured Features

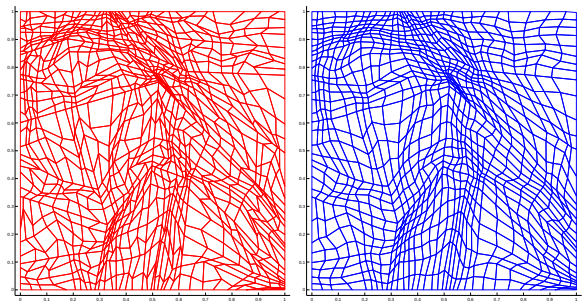
Milan Kucharik, kucharik@lanl.gov
Mikhail Shashkov, shashkov@lanl.gov

In numerical simulations of fluid flow, the choice of the computational grid is crucial. Traditionally, there have been two viewpoints, utilizing the Lagrangian or the Eulerian framework, each with its own advantages and disadvantages. In a pioneering paper [1], Hirt et al. developed the formalism for a grid whose motion could be determined as an independent degree of freedom, and showed that this general framework could be used to combine the best properties of Lagrangian and Eulerian methods. This class of methods has been termed Arbitrary Lagrangian-Eulerian or ALE.

It is most usual to separate the ALE algorithm into three individual phases. These are: 1) a Lagrangian phase, in which the solution and grid are updated, 2) a rezoning phase, in which the nodes of the computational grid are moved to a more optimal position, and 3) a remapping phase, in which the Lagrangian solution is interpolated onto the rezoned grid. Here, we focus to the second (rezoning) stage of the ALE algorithm, producing more optimal and smoother computational mesh for the next Lagrangian step. We are looking for a mesh optimization method applicable to logically-orthogonal quadrilateral computational meshes, allowing to involve several unstructured features, such as material interfaces, virtual parts of the mesh, central points, and slide-lines. One of candidates for such rezoner if the Reference-Jacobian (RJ) mesh smoothing method [2].

RJ Method

In the first step of the RJ mesh smoothing method, locally optimal position (reference position according to the local condition number) for each mesh node is found. Then, in the second step, global optimization of objective function measuring the (least-square) distance between Jacobians of the rezoned mesh and the optimal reference mesh. RJ smoothing preserves Lagrangian shape of the mesh, and smoothes the mesh by minor nodal movement, as shown in the following figure. This method is applicable to smoothing of

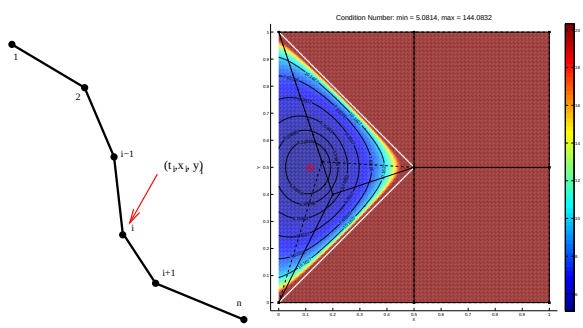


Disturbed (red) and smoothed (blue) mesh. Both meshes remain very close to each other, smoothed mesh has much higher quality.

general meshes in 2D and 3D, and is suitable for ALE method.

Material Interfaces

On boundary/interface, the mesh nodes are allowed to move only on the broken line of the original boundary. In the following figure, one pa-



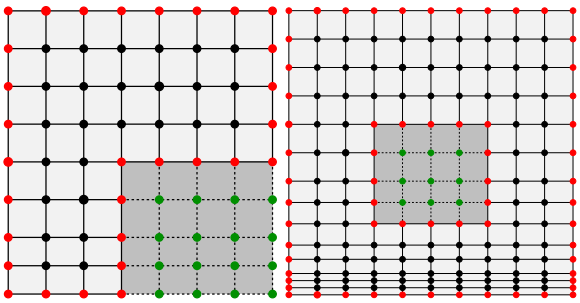
Material interface/boundary parameterization and condition number colormap for one particular boundary vertex, with original perturbed (solid) and new smoothed (dashed) mesh.

rameterized boundary segment and one example of smoothed boundary node along the segment

are presented. Parameterization is defined as distance along the broken line from the segment beginning.

Virtual Mesh Regions

To introduce unstructured features to the structured quadrilateral mesh, we consider virtual cells and virtual nodes (dark-gray cells and green nodes in the following figure). Virtual cells are



L-shape mesh and mesh with hole. Virtual cells marked in dark gray, virtual nodes in green, boundary/interface nodes in red.

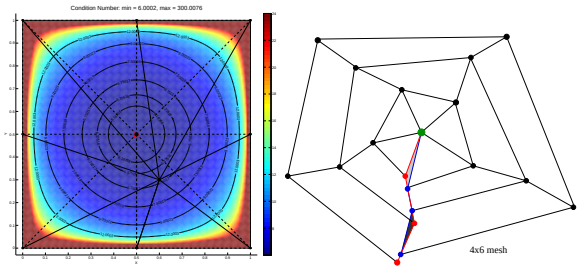
not considered for local nor global mesh quality functional evaluation. Nodes on the interface between virtual and real mesh regions (red) are treated by the same parameterization as interface/boundary nodes. This technique allows to construct L-shaped (and other shaped) computational meshes, or meshes with holes.

Center-Points and Slide-Lines

Center-points and slide-lines represent another possibility how to involve unstructured features in the structured computational mesh. Examples are shown in the following figure.

Center-point is defined as several logical mesh nodes sharing the same physical location. They are "glued" together and are allowed to move only as one point. In the first subfigure, we can see convergence of the center-point to the correct minimum of the local condition number. In the second subfigure, the center-point is marked in green.

Slide-line is defined as two different edge segments, which are only allowed to slide along each



Colormap of condition number for center-point and original perturbed (solid) and smoothed (dashed) mesh. Schematic 4×6 mesh with (green) center-point and slide-line including master (red) and slave (blue) segments. Hole (light-gray) and overlap (dark-gray) are highlighted.

other. They are presented as a master (red) and slave (blue) segments. For nodal movement, parameterization of the master line is used for both master and slave nodes. This process can be used for either connecting of two different mesh parts, or for closing of logically circular computational mesh. Overlappings and holes can appear in the slide-line region.

All the mentioned techniques allow construction of topologically complex meshes using standard, logically orthogonal mesh representation. Our implementation of the RJ technique is able to manage all the mesh features, and provides high-quality smoothed meshes suitable for ALE methods.

Acknowledgements

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References

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[2] P. M. Knupp, L. G. Margolin, and M. J. Shashkov. Reference Jacobian optimization-based rezone strategies for arbitrary Lagrangian Eulerian methods. *J. Comput. Phys.*, 176(1):93–128, 2002.