

## J. David Moulton

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### Education

- B.Eng. Physics, McMaster University, *summa cum laude*, 1988
- M.Eng. Physics, McMaster University, 1990  
*Thesis: Diffusion Modelling of Picosecond Pulse Propagation in Turbid Media*  
(August 1990)
- Ph.D. Mathematics, University of British Columbia, 1997  
*Thesis: Nodal Methods: Performance, Analysis and Fast Iterative Solvers*  
(November 1996)

### Research Experience

➤ **Scientist-4**

*Team Leader: Mathematical Modeling and Uncertainty Quantification*

*October 2008 – present*

Applied Mathematics and Plasma Physics, Theoretical Division, Los Alamos National Laboratory  
Research: multilevel upscaling and homogenization, mixed finite elements, mimetic and compatible finite difference discretizations, sensitivity analysis, uncertainty quantification, large-scale parallel and hybrid computing, Krylov subspace and multigrid solvers.

➤ **Technical Staff Member**

*October 1998 – September 2008*

Mathematical Modeling and Analysis, Theoretical Division, Los Alamos National Laboratory  
Research: multilevel upscaling and homogenization of diffusive processes, mixed FEMs, parallel computing and cluster design, Krylov subspace and multigrid solvers.

➤ **Postdoctoral Research Associate**

*November 1996 – September 1998*

Center for Nonlinear Studies and T-7, Theoretical Division, Los Alamos National Laboratory  
Research: Homogenization, Mixed-Hybrid FEMs, Krylov subspace and multigrid solvers.

➤ **Research Assistant**

*September 1995 – August 1996*

Institute of Applied Mathematics, University of British Columbia  
Research: Mixed FEM discretization of elliptic PDEs with highly discontinuous coefficients.

➤ **Student Visitor**

*October 1994, March 1995*

T-7, Theoretical Division, Los Alamos National Laboratory  
Research: Multigrid and Krylov subspace iterative solution of Nodal Methods.

## Research Interests

- Discretization techniques for partial differential equations, particularly elliptic equations with discontinuous coefficients. These include mixed finite element and mimetic finite difference methods for flow in porous media, and nodal methods for neutron diffusion in reactors.
- Iterative methods such as multigrid and multigrid preconditioning of Krylov subspace algorithms for the solution of the discrete linear systems that arise from the aforementioned discretizations. Typically these linear systems have a non-standard sparsity pattern and may be indefinite.
- The development of new, numerical, multilevel upscaling techniques that will facilitate the accurate coarse-scale numerical treatment of flow through highly heterogeneous media.
- The Bayesian framework for statistical inference in inverse problems, state and parameter estimation and uncertainty quantification. Development of new adaptive and multilevel Markov Chain Monte Carlo (MCMC) samplers for exploration of high-dimensional multimodel posterior distributions.
- Using forward and adjoint sensitivity analysis to study parametric sensitivity in high-fidelity models of atmospheric flows, such as hurricane intensification. Development of second-order adjoint methods for targeting strategies and data assimilation methods.
- Parallel computing, and in particular, parallel iterative solvers. Many iterative solvers have inherently serial components that make their parallelization difficult. Multilevel upscaling may provide some insight into alternative approaches.

## Computing Experience

- **Numerical Analysis:** 22 years experience in programming algorithms for the numerical solution of ODEs and PDEs. Focusing on PDEs, discretization techniques include, finite difference, finite element (FE), Mixed-Hybrid FE and finite volume methods. Iterative solution techniques include preconditioned Krylov subspace methods and multigrid.
- **Programming:** Experience with many languages and tools, including Fortran 77/90/95, C/C++, MPI, Python, Perl, matlab, Maple,  $\text{\TeX}$ ,  $\text{\LaTeX}$ .
- **System Administration:** 15 years experience in the administration and maintenance of heterogeneous computer networks comprised of Unix workstations, X-terminals, and MS-Windows based PCs.
- **Participant:** DOE Computational Science Workshop, Summer 1992.

## Scholarships, Fellowships and Awards

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|---------|--|
| 05/2012 | Postdoctoral Distinguished Mentor Award 2011, Los Alamos National Laboratory   |
| 06/2004 | Teamwork Award, Los Alamos National Laboratory<br><i>Implementation and demonstration at scale (3584 processors, 3.5 billion unknowns) of an MPI-based parallel multigrid solver, BoxMG.</i> |
| 09/2002 | Achievement Award, Los Alamos National Laboratory<br><i>Outstanding contributions to the summer student program.</i>   |
| 09/1998 | Achievement Award, Los Alamos National Laboratory<br><i>Designing and building the Beowulf class supercomputer “Avalon”.</i>   |
| 10/1997 | High Performance Computing Fellowship, Los Alamos National Laboratory  |
| 10/1996 | High Performance Computing Fellowship, Los Alamos National Laboratory  |
| 09/1993 | Graduate Fellowship, University of British Columbia  |
| 09/1992 | Graduate Fellowship, University of British Columbia  |
| 09/1991 | Postgraduate Fellowship, Natural Sciences and Engineering Research Council of Canada   |
| 09/1991 | Graduate Fellowship Supplement, University of British Columbia   |
| 09/1990 | Postgraduate Fellowship, Natural Sciences and Engineering Research Council of Canada   |
| 09/1990 | Graduate Fellowship Supplement, University of British Columbia   |
| 09/1989 | Ontario Graduate Scholarship   |
| 09/1988 | Centennial Graduate Scholarship, McMaster University   |
| 05/1988 | Harold Johns’ Medical Physics Student Fellowship   |
| 05/1987 | Undergraduate Research Award, Natural Sciences and Engineering Research Council of Canada  |
| 09/1984 | Chancellors’ Admission Scholarship, McMaster University  |

## Academic Activities

- **Organizing Committee:** *Copper Mountain Conference on Multigrid Methods*, Biennially in Copper Mountain, CO, member since 2002. Co-editor of the conferences’ special issues of *Numerical Linear Algebra with Applications*, since 2003.
- **Co-Organizer:** *Transforming Water Resource Management with Open-Source Community Tools*, Special Session, Computational Methods in Water Resources, XIX International Conference University of Illinois at Urbana-Champaign, Urbana, IL, June 17–21, 2012.
- **Co-Organizer:** *Bridging Applied Mathematics and Computational Geosciences for Environmental Management*, Minisymposium, Seventh International Congress on Industrial and Applied Mathematics, Vancouver, BC, Canada, July 19–21, 2011.
- **Co-organizer:** *Practical Scientific Computing in Python*, Hands on workshop at the Center for Non-linear Studies, Los Alamos National Laboratory, Los Alamos NM, June 10 – 11, 2005. Instructors: John Hunter (U of Chicago) and Fernando Pérez (CU Boulder).

- **Organizer:** *Setting up and Using a Small Linux Cluster*, Short course at the SIAM Annual Meeting, Town and Country Resort, San Diego, CA, July 9–13, 2001.
- **Co-organizer:** *Third Annual ASCI Tri-Lab Workshop on Solvers*, Los Alamos Inn, Los Alamos, NM, December 1–2, 1999.
- **Co-organizer:** *Multiscale Modeling and Simulation of Flow and Transport in Porous Media*, University of New Mexico, Los Alamos Campus, Los Alamos, NM, August 11–13, 1999.
- **Organizer:** *Volume Averaging Methods in Porous Media – Short Course*, Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, NM, July 6–10, 1998.  
Instructor: Dr. Stephen Whitaker of UC Davis,
- **Organizer:** *Influence and Treatment of Multiple Scales in Modeling Heterogenous Media*, Minisymposium, SIAM Annual Meeting, Stanford University, CA, July 14-18, 1997.
- **Journal Referee:** SIAM J. Num. Anal., SIAM J. Sci. Comp., Numer. Lin. Alg. App., App. Numer. Math., J. Comp. Phys., Rev. Geophys., and, Water Resour. Res.
- **Proposal Reviews:** NSF(USA), DOE/Office of Science (USA), NSERC (Canada), Internal LDRD/ER
- **Member:** Society of Industrial and Applied Mathematics, American Mathematical Society, Society of Petroleum Engineers, American Geophysical Union

## Selected Presentations

- [1] *Exploring parallelization and performance of a Particle in Cell method on curvilinear grids*. Copper Mountain Conference on Iterative Methods, Copper Mountain, CO, April 6 – 11, 2014.
- [2] *Hierarchical testing with automated document generation for Amanzi (ASCEM's subsurface flow and reactive transport simulator)*. AGU 2013 Fall Meeting, December 9 – 13, 2013.
- [3] *Using manycore and accelerated architectures to improve performance of robust structured multigrid methods*. Copper Mountain Conference on Multigrid Methods, Copper Mountain, CO, March 17 – March 22, 2013.
- [4] *A flexible and extensible multi-process simulation capability for the terrestrial arctic*. Frontiers in Computational Physics *Modeling the Earth System*, Boulder, CO, December 16 – 20, 2012.
- [5] *Amanzi: a parallel open-source flow and reactive-transport simulator for environmental applications*. Computational Methods in Water Resources, the XIX International Conference, University of Illinois at Urbana-Champaign, Urbana, IL, June 17 – 21, 2012.
- [6] *Advances in multilevel model reduction for reservoir simulation*. Interpore 2012, Purdue University, West Lafayette, IN, May 14 – 16, 2012.
- [7] *Amanzi: a parallel open-source flow and reactive-transport simulator for environmental applications*. Interpore 2012, Purdue University, West Lafayette, IN, May 14 – 16, 2012.
- [8] *Amanzi and Akuna: Two new community codes for subsurface contaminant flow and transport*. AGU Fall Meeting 2011, San Francisco, CA, December 5–9, 2011.
- [9] *Using advanced discretizations for modeling subsurface flow on highly distorted unstructured grids*. 7<sup>th</sup> International Congress on Industrial and Applied Mathematics - ICIAM 2011, Vancouver, BC, Canada, July 18 – 22, 2011.

- [10] *Recent advances in robust structured multigrid methods.* Advances in the Numerical Solution of Constrained Differential Equations, University of British Columbia, Vancouver, BC, Canada, July 15 – 17, 2011.
- [11] *Robust and adaptive multigrid methods: Comparing structured and algebraic approaches.* Copper Mountain Conference on Multigrid Methods, Copper Mountain, CO, March 27 – April 1, 2011.
- [12] *Overview of the multi-process high performance simulator.* ASCEM-NEAMS Workshop, Buffalo Thunder, Santa Fe, NM, February 17–18, 2011.
- [13] *The multi-process high performance simulator: Amanzi.* Advanced Subsurface Capability for Environmental Management (ASCEM) PI Meeting, Buffalo Thunder Resort, Santa Fe, NM, February 15–16, 2011.
- [14] *The multi-process high performance computing simulator thrust.* Advanced Subsurface Capability for Environmental Management (ASCEM) Kickoff Meeting, Lawrence Berkeley National Laboratory, Berkeley, CA, January 20 – 21, 2010.
- [15] *Challenges for multilevel sampling in statistical inference.* Copper Mountain Conference on Multigrid Methods, Copper Mountain, CO, March 22 – 27, 2009.
- [16] *Multilevel modeling in highly heterogeneous media.* SIAM Conference on Computational Science and Engineering, Miami Hilton Hotel, Miami, FL, March 2–6, 2009.
- [17] *Uncertainty and sensitivity in multilevel hierarchical modeling.* Poster at the ASCR Applied Mathematics Research Principal Investigators Meeting, Argonne National Laboratory, Argonne, IL, October 15–17, 2008.
- [18] *Multilevel modeling: Multigrid's lost twin.* Computational Infrastructure for Geodynamics (CIG) Workshop on Mathematical and Computational Issues in the Solid Earth Geosciences, Santa Fe, NM, September 15-17, 2008.
- [19] *Multilevel modeling of heterogeneous media and implications for sample-based inversion.* Scientific Computing and Applied/Industrial Mathematics seminar series, University of British Columbia, Vancouver, BC and Centre for Scientific Computing Simon Fraser University, Burnaby, BC, June 20/25, 2008.
- [20] *A multilevel multiscale mimetic ( $M^3$ ) method for two-phase flows in porous media.* Copper Mountain Conference on Iterative Methods, Copper Mountain, CO, April 6–11, 2008.
- [21] *Multilevel approximations in sample-based inversion from the dirichlet-to-neumann map.* First International Congress of IPIA Conference on Applied Inverse Problems 2007: Theoretical and Computational Aspects, University of British Columbia, Vancouver, BC, Canada, June 25 – 29, 2007.
- [22] *Multilevel upscaling in multiscale modeling and parameter estimation.* Poster at the ASCR Applied Mathematics Research Principal Investigators Meeting, Terascale Simulation Facility, Lawrence Livermore National Laboratory, Livermore, CA, May 22 – 24, 2007.
- [23] *Revisiting augmented flux-space preconditioning of mimetic discretizations.* Copper Mountain Conference on Multigrid Methods, Copper Mountain, CO, March 18 – 23, 2007.
- [24] *An efficient nonlinear solver for a mesh smoothing problem.* Los Alamos Computer Science Institute (LACSI), Eldorado Hotel, Santa Fe, NM, October 11–13, 2005.

- [25] *Towards flexible multilevel upscaling of parameters and models*. Invited talk at the Canadian Applied and Industrial Mathematics (CAIMS) Society Annual Meeting, Winnipeg, MN, June 16–18, 2005.
- [26] *Multilevel upscaling: Multigrid's lost twin*. Copper Mountain Conference on Multigrid Methods, Copper Mountain, CO, April 3–8, 2005.
- [27] *Multilevel upscaling through variational coarsening*. SIAM Conference on Computational Science and Engineering, Disney's Coronado Springs Resort, Orlando, FL, February 12–15, 2005.
- [28] *Mimetic preconditioning of mixed-hybrid discretizations*. Los Alamos Computer Science Institute (LACSI), Eldorado Hotel, Santa Fe, NM, October 12–14, 2004.
- [29] *Mimetic preconditioners for mixed discretizations of the diffusion equation*. Poster at the IMA Hot Topics Workshop: Compatible Spatial Discretizations for Partial Differential Equations, IMA, University of Minnesota, MN, May 11–15, 2004.
- [30] *Performance tuning of parallel structured multigrid*. Copper Mountain Conference on Iterative Methods, Copper Mountain, CO, March 28 – April 2, 2004.
- [31] *Performance tuning of structured black box multigrid for massively parallel distributed hierarchical memory architectures*. SIAM Conference on Parallel Processing for Scientific Computing, Hyatt at Fisherman's Warf, San Francisco, CA, February 25–27, 2004.
- [32] *New results in multilevel upscaling or homogenization*. SIAM Annual Meeting/First Joint meeting of CAIMS and SIAM, Fairmont The Queen Elizabeth Hotel, Montreal, QC, June 16–20,, 2003.
- [33] *A comparison of mimetic and variational preconditioners for mixed-hybrid discretizations of the diffusion equation*. Copper Mountain Conference on Multigrid Methods, Copper Mountain, CO, March 30 – April 4,, 2003.
- [34] *A comparison of mimetic and variational preconditioners for mixed and mixed-hybrid discretizations*. SIAM Conference on Mathematical and Computational Issues in the Geosciences, Radisson Hotel and Suites Austin, Austin, TX, March 17–20,, 2003.
- [35] *A practical guide to building and using beowulf (commodity) clusters*. Half-day workshop at the 2001 ASME International Mechanical Engineering Congress & Exposition, Hilton/Sheraton New York Hotel and Towers, New York, NY, November 11–16,, 2001.
- [36] *An augmented systems approach to mimetic preconditioning*. Copper Mountain Conference on Multigrid Methods, Copper Mountain, CO, April 1–6,, 2001.
- [37] *An introduction to beowulf-class clusters*. Invited talk, Computer Science Department, University of British Columbia, Vancouver, BC, Canada, July 27,, 1999.
- [38] *Parallel multigrid homogenization on workstation clusters*. Copper Mountain Conference on Multigrid Methods, Copper Mountain, CO, April 11–16,, 1999.
- [39] *Relaxation algorithms for cell- and edge-based discretizations*. Copper Mountain Conference on Iterative Methods, Copper Mountain, CO, March 30 – April 3,, 1998.
- [40] *Multigrid homogenization of heterogeneous porous media*. Colorado Days, University of Colorado at Boulder, Boulder, CO, April 30 – May 2,, 1998.

## Selected Publications

- [1] G. L. Delzanno, E. Camporeale, J. D. Moulton, J. E. Borovsky, E. MacDonald, and M. F. Thomsen. CPIC: A Curvilinear Particle-in-Cell Code for Plasma-Material Interaction Studies. *IEEE T. Plasma Sci.*, 41:3577–3587, 2013.
- [2] X. He, L. Jiang, and J. D. Moulton. A stochastic dimension reduction multiscale finite element method for groundwater flow problems in heterogeneous random porous media. *J. Hydrol.*, 478:77–88, 2013.
- [3] S. L. Painter, J. D. Moulton, and C. J. Wilson. Modeling challenges for predicting hydrologic response to degrading permafrost. *Hydrogeol. J.*, 21(1):221–224, 2013.
- [4] L. Jiang, J. D. Moulton, and J. Wei. A hybrid HDMR for mixed multiscale finite element method with application for flows in random porous media. Technical Report LA-UR 12-26499, Applied Mathematics and Plasma Physics Group, Los Alamos National Laboratory, 2012. *to appear in SIAM Multiscale Modeling and Simulation*.
- [5] M. Freshley, S. Hubbard, I. Gorton, J. D. Moulton, C. I. Steefel, V. Freedman, H. Wainwright, et al. Phase II Demonstration. Technical Report ASCEM-SITE-2012-01 (LANL: LA-UR 12-25496), Office of Environmental Management, United States Department of Energy, Washington, DC, 2012.
- [6] M. L. Porter, E. Coon, Q. Kang, J. D. Moulton, and W. J. Carey. Multicomponent interparticle-potential lattice Boltzmann model for fluids with large kinematic viscosity ratios. *Phys. Rev. E*, 86(3):036701, 2012.
- [7] S. P. MacLachlan, J. D. Moulton, and T. P. Chartier. Robust and adaptive multigrid methods: comparing structured and algebraic approaches. *J. Numer. Lin. Alg. App.*, 19:389 – 413, 2012.
- [8] L. Jiang, J. D. Moulton, and D. Svyatskiy. Analysis of stochastic mimetic finite difference methods and their applications in single-phase stochastic flows. *Comput. Methods Appl. Mech. Engrg.*, 217:58–76, 2012.
- [9] D. Copeland, L. Jiang, and J. D. Moulton. Expanded mixed multiscale finite element methods and their applications for flows in porous media. *Multiscale Model. Sim.*, 10(2):418–450, 2012.
- [10] J. D. Moulton, J. C. Meza, M. W. Buksas, M. Day, et al. High-level design of Amanzi, the multi-process high performance computing simulator. Technical Report ASCEM-HPC-2011-03-1 (LANL: LA-UR 12-22193), Office of Environmental Management, United States Department of Energy, Washington, DC, 2012.
- [11] H. C. Godinez and J. D. Moulton. An efficient matrix-free algorithm for the ensemble kalman filter. *Computat. Geosci.*, 16(3):565 – 575, 2012.
- [12] G. Srinivasan, E. Keating, J. D. Moulton, Z. Dash, and B. Robinson. Convolution-based particle tracking method for transient flow. *Computat. Geosci.*, 16(3):551–563, 2012.
- [13] K. Lipnikov, J. D. Moulton, and D. Svyatskiy. Adaptive strategies in the Multilevel Multiscale Mimetic ( $M^3$ ) method for two-phase flows in porous media. *Multiscale Model. Sim.*, 9(3):991–1016, 2011.
- [14] D. Higdon, C. S. Reese, J. D. Moulton, J. A. Vrugt, and C. Fox. Posterior exploration for computationally intensive forward models. Technical report. Chapter 16, in *The Handbook of Markov Chain Monte Carlo*, Eds. X.-L. Meng, A. Gelman, and G. Jones, Chapman and Hall/CRC, 2011.
- [15] J. E. Dendy and J. D. Moulton. Black box multigrid with coarsening by a factor of three. *J. Numer. Lin. Alg. App.*, 17:577–598, 2010.

- [16] S. Hubbard, M. Freshley, D. Moulton, P. Lichtner, J. Bell, et al. ASCEM Phase I Demonstration. Technical Report ASCEM-SITE-102010-1, Office of Environmental Management, United States Department of Energy, Washington, DC, 2010.
- [17] K. Lipnikov, J. D. Moulton, and D. Svyatskiy. A multiscale multilevel mimetic ( $M^3$ ) method for well-driven flows in porous media. *Procedia Computer Science*, 1(1):771 – 779, 2010.
- [18] E. T. Coon, S. P. MacLachlan, and J. D. Moulton. Local post-processing for locally conservative fluxes in the galerkin method for groundwater flows. Technical Report LA-UR 09-08292, Applied Mathematics and Plasma Physics Group, Los Alamos National Laboratory, 2009.
- [19] K. Lipnikov, J. D. Moulton, and D. Svyatskiy. A multilevel multiscale mimetic ( $M^3$ ) method for an anisotropic infiltration problem. In G. Allen, J. Nabrzyski, E. Seidel, G. D. van Albada, J. J. Dongarra, and P. M. A. Sloot, editors, *Computational Science–ICCS 2009, 9<sup>th</sup> International Conference Baton Rouge, LA, May 25-27, 2009. Proceedings, Part I*, volume 5544 of *Lecture Notes in Computer Science*, pages 685–694. Springer-Verlag, 2009.
- [20] K. Lipnikov, J. D. Moulton, and D. Svyatskiy. A Multilevel Multiscale Mimetic ( $M^3$ ) method for two-phase flows in porous media. *J. Comput. Phys.*, 227(14):6727–6753, 2008.
- [21] M. Berndt, G. Hansen, and J. D. Moulton. Efficient nonlinear solvers for Laplace-Beltrami smoothing of three-dimensional unstructured grids. *Comput. Math. Appl.*, 55(12):2791–2806, 2008.
- [22] J. D. Moulton, C. Fox, and D. Svyatskiy. Multilevel approximations in sample-based inversion from the Dirichlet-to-Neumann map. *J. Phys. Conf. Ser.*, 124:012035 (10pp), 2008.
- [23] J. D. Moulton, T. M. Austin, M. Shashkov, J. E. Morel, and D. Svyatskiy. A mimetic preconditioner for mixed discretizations of the diffusion equation. Technical Report LA-UR 07-8396, Mathematical Modeling and Analysis Group, Los Alamos National Laboratory, 2007.
- [24] D. Moulton, K. Lipnikov, J. Fung, and S. Runnels. Discretization schemes on polygonal and polyhedral grids for diffusion problems. Technical Report LA-UR 07-1588, Mathematical Modeling and Analysis Group, Los Alamos National Laboratory, 2007. *to appear in the Proceedings of NECDC 2006, October 23–27, Los Alamos National Laboratory, Los Alamos, NM.*
- [25] S. P. MacLachlan and J. D. Moulton. Multilevel upscaling through variational coarsening. *Water Resour. Res.*, 42, 2006.
- [26] T. M. Austin, M. Berndt, and J. D. Moulton. A memory efficient parallel tridiagonal solver. Technical Report LA-UR 03-4149, Mathematical Modeling and Analysis Group, Los Alamos National Laboratory, Los Alamos, NM, 2004.
- [27] M. Berndt, K. Lipnikov, J. D. Moulton, and M. J. Shashkov. Convergence of mimetic finite difference discretizations of the diffusion equation. *East–West Journal of Numerical Mathematics*, 9:253–316, 2001.
- [28] D. M. Tartakovsky, J. D. Moulton, and V. A. Zlotnik. Kinematic structure of minipermeameter flow. *Water Resour. Res.*, 36:2433–2442, 2000.
- [29] J. E. Dendy and J. D. Moulton. Some aspects of multigrid for mixed discretizations. In E. Dick, K. Rienslagh, and J. Vierendeels, editors, *Multigrid VI, Proceedings of the Sixth European Multigrid Conference, held in Gent, Belgium, September 27-30, 1999*, volume 14 of *Lecture Notes in Computational Science and Engineering*, pages 80–86. Springer-Verlag, 2000.



- [30] J. David. Moulton, Stephan Knappek, and Joel E. Dendy. Multilevel upscaling in heterogeneous porous media. Research Highlights LA-UR 99-4754, Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, NM, January 1999.
- [31] J. D. Moulton, J. E. Dendy, and J. M. Hyman. The black box multigrid numerical homogenization algorithm. *J. Comput. Phys.*, 141:1–29, 1998.
- [32] J. D. Moulton, J. E. Morel, and U. M. Ascher. Approximate schur complement preconditioning of the lowest-order nodal discretizations. *SIAM J. Sci. Comput.*, 19(1):185–205, Jan 1998.
- [33] M. S. Patterson, J. D. Moulton, B. C. Wilson, K. W. Berndt, and J. R. Lakowicz. Frequency-domain reflectance for the determination of the scattering and absorption properties of tissue. *Appl. Opt.*, 30(31):4474–4476, 1991.
- [34] M. S. Patterson, S. J. Madsen, J. D. Moulton, and B. C. Wilson. Diffusion equation representation of photon migration in tissue. In G. L. Hieter, editor, *International Microwave Symposium*, volume 2, pages 905–908. IEEE MTT-S, 1991.
- [35] S. J. Madsen, M. S. Patterson, B. C. Wilson, Y. D. Park, J. D. Moulton, S. L. Jacques, and Y. Hefetz. Time resolved diffuse reflectance and transmittance studies in tissue simulating phantoms: A comparison between theory and experiment. In Britton Chance, editor, *Time Resolved Spectroscopy and Imaging of Tissues*, volume 1431, pages 42–51. SPIE, 1991.
- [36] M. S. Patterson, J. D. Moulton, B. C. Wilson, and B. Chance. Applications of time-resolved light scattering measurements to photodynamic therapy dosimetry. In T. J. Dougherty, editor, *Photodynamic Therapy Mechanisms II*, volume 1203, pages 62–75. SPIE, 1990.
- [37] B. C. Wilson, M. S. Patterson, S. T. Flock, and J. D. Moulton. The optical absorption and scattering properties of tissues in the visible and near-infrared wavelength range. In R. H. Douglas, J. Moan, and F Dall’Acqua, editors, *Light in Biology and Medicine*, volume 1, pages 45–52. Plenum Publishing Corp., 1988.