Multigrid Homogenization of Heterogenous Porous Media

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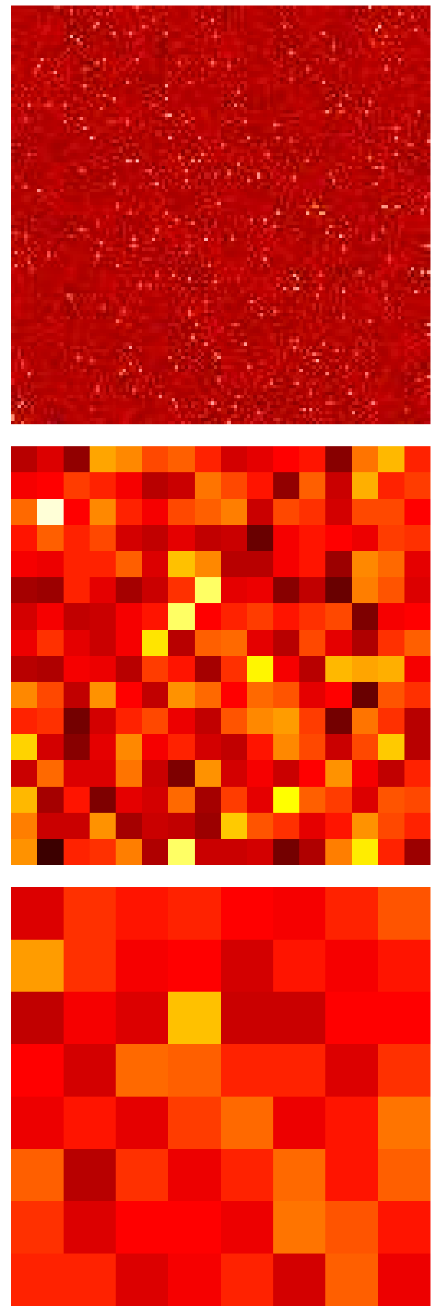
Introduction:

The earth is perhaps the most obvious example of a porous medium. Flows in porous media are part of our everyday lives, from water in the garden, to oil in underground reservoirs. The mathematical modeling of flow in porous media, particularly with the ever increasing power of computers, is playing a fundamental and increasingly important role in the forecasting of petroleum reservoir performance, groundwater supply, and subsurface contaminant flow. A critical underlying problem in the numerical treatment of these models is the need to resolve the multiscale structure of heterogeneous geological formations. Unfortunately, the length scales observed in sedimentary laminae range from the millimeter scale upward, while the simulation domain may be on the order several kilometers. As a result, fully resolved simulations are computationally intractable, and yet the fine-scale variations of the model's parameters (e.g., structure and orientation of laminae) significantly affect the coarse-scale properties of the solution (e.g., average flow rates). This complex interaction of significantly different length scales is not unique to flows in porous media, but arises in many other disciplines, and is currently studied by T-7 in several other important contexts; including composite materials, global atmospheric and ocean circulation models, and in solid-solid phase transitions.

Homogenization or Upscaling:

The objective of a homogenization or upscaling procedure is to define an approximate mathematical model in which the *effective* properties of the medium vary on a coarse scale suitable for efficient computation, while preserving certain coarse-scale properties of the fine-scale solution (e.g., average flow rates). To homogenize



A randomly generated permeability field that exhibits variations over several orders of magnitude is shown in the top image. Brighter colors indicate a higher permeability. MGH has been applied to this data to generate two successively coarser representations of the permeability that are suitable for numerical simulation (the middle and bottom images).

or upscale heterogeneous media, the macroscopic flow model, which contains parameters that vary on the microscopic or fine-scale, is *averaged*, in some sense, over the microscopic length scales.

For some material properties, upscaling is trivial. For example, the average porosity or saturation of a volume can be accurately approximated by the arithmetic average of all the porosities or saturations within that volume. However, this simplistic approach fails for other medium characteristics, such as the effective permeability, which is the fundamental parameter in single-phase saturated flow modeling. Even when fine-scale variations in the permeability are small relative to the scale of the homogenized cell volume, the geostatistical arithmetic, geometric or harmonic averages of the fine-scale permeability can differ by orders of magnitude from the true effective permeability. In addition, the increasing use of geostatistical techniques to infer physically meaningful fine-scale realizations of heterogeneous geological formations from sparse and inherently multi-scale measurement data demands more accurate and efficient homogenization procedures.

Multigrid:

The numerical treatment of a mathematical model ultimately relies on the solution of a system of discrete, often linear, equations. Although, in principle, direct methods could be used to solve such a system, this is computationally intractable for large two- and three-dimensional problems. Instead computational efficiency is achieved with iterative solution algorithms such as multigrid.

Multigrid methods gained recognition in the late 1970's as an efficient algorithm for the solution of the discrete linear systems that arise from models of diffusive phenomena (e.g., heat conduction, neutron diffusion, single-phase saturated flow). These methods achieve their efficiency through the recursive use of successively coarser discrete problems (i.e., a sequence of coarse-grid discrete operators). Unfortunately, early multigrid algorithms were fragile, their efficiency strongly dependent on the variability of the model's coefficients. Considerable research in the early 1980's, much of it in T-7, led to the first multigrid algorithms that could be used reliably for a large class of practical problems. The key to the success of these robust *Black Box* methods was the use of the fine-scale discrete model to construct, through a variational principle, the successively coarser coarse-grid operators.

Multigrid Homogenization:

Currently, T-7 is actively involved in both the modeling of multiscale phenomena and the development of multilevel iterative solution algorithms. The development of the multigrid homogenization (MGH) algorithm for single-phase saturated flows was motivated by the observation that equivalent multi-scale issues arise in both fields. In particular, the robustness and efficiency of the Black Box multigrid method strongly suggested that its variational coarsening procedure produced excellent coarse-grid operators. Thus, it was hypothesized that the coarsening procedure implicit to Black Box multigrid could be interpreted as a homogenization procedure that produced a coarse-scale discrete model, from which the coarse-scale model parameters (i.e., the permeability) could be extracted.

This interpretation led to the MGH algorithm [1], which has demonstrated its efficiency and accuracy for fine-scale periodic media in two dimensions. We are currently investigating its application to general fine-scale structures in two- and three-dimensional media. In addition, we are confident that this interpretation will lead to further improvements in the variational coarsening procedure, and hence, to the Black Box multigrid algorithm.

References

[1] J. D. Moulton, J. E. Dendy, and J. M. Hyman. The black box multigrid numerical homogenization algorithm. *J. Comput. Phys.*, 141:1–29, 1998.

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