

Material order independent interface reconstruction using power diagrams

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We have developed a novel method for interface reconstruction in multi-material volume-of-fluid (VOF) flow simulations. It uses a weighted Voronoi diagram, known as a power diagram, and approximate calculation of a local center of mass for each material. The method has no material order dependence, is completely general, working for an arbitrary number of materials on arbitrary polygonal grids, provides a decomposition of a cell into convex, pure material regions, is equivalent to Youngs' method for two-material simulations, and is naturally extendable to 3D.

In VOF or volume tracking simulations, the interfaces between materials are not explicitly tracked, rather the volume fraction, $0 \leq f \leq 1$, of each material in a cell is tracked. For pure cell, the volume fractions will all be either zero or one. In a mixed cell, multiple materials will have volume fractions strictly between zero and one. When the interface in a mixed cell is needed, it is reconstructed based on the volume fraction data. Piecewise linear interface calculation methods [2] calculate a normal direction for the material interface then match a line with that normal such that it cuts off the appropriate volume for each material in the cell.

When a mixed cell contains $N_m > 2$ materials, PLIC methods traditionally work by separating one material from all the remaining $N_m - 1$ materials, then recalculating the volume fractions for the remaining $N_m - 1$ materials in the unused portion of the cell, and then separating the next material from the $N_m - 2$ remaining materials and so on. The resulting reconstruction will depend on the order in which the materials are processed.

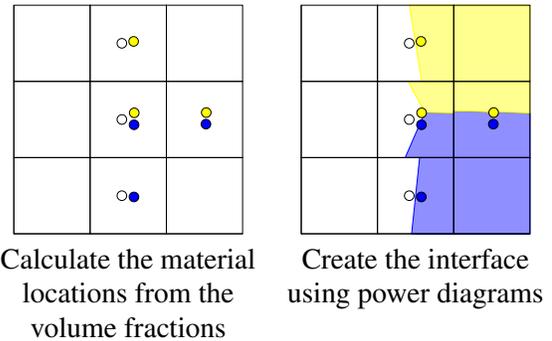


Figure 1: Steps in power diagram based reconstruction

Simulations have little or no guidance available for selecting a material ordering and typically rely on the user to specify an ordering *a priori*.

Our power diagram method follows a different approach as shown in Figure 1. First, an approximate center of mass for each material in a cell is derived from the volume fraction data. Then a *power diagram* is created using these points as generators with weights adjusted to match the volume fractions.

A power (or Laguerre) diagram is similar to a Voronoi diagram but the generators have an associated weight [1]. The Laguerre distance to a generator, \mathbf{x}_i , is defined as

$$d_L^2(\mathbf{x}, \mathbf{x}_i) = d^2(\mathbf{x}, \mathbf{x}_i) - \omega_i \quad (1)$$

A Power diagram cell is the set of points

$$cell((\mathbf{x}_i, \omega_i)) = \{\mathbf{x} \in \mathbb{R}^n \mid d_L^2(\mathbf{x}, \mathbf{x}_i) < d_L^2(\mathbf{x}, \mathbf{x}_j), \\ j = 1, \dots, N, j \neq i\} \quad (2)$$

The weights associated with each point generator are adjusted to ensure that each power diagram cell, once clipped to the mesh cell, has the required volume fraction.

The calculation of the point generators begins by treating the volume fractions of a material like a density and calculating a linear reconstruction of the volume fraction data. The reconstruction has the form

$$\tilde{f}(\mathbf{x}) = f + \delta \cdot (\mathbf{x} - \mathbf{x}_c(\Omega)) \quad (3)$$

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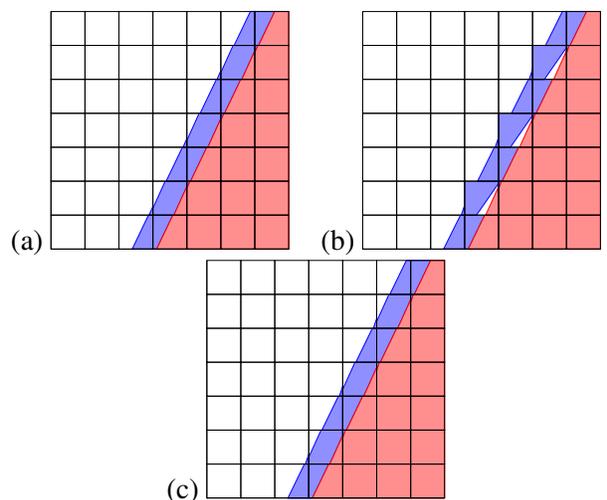


Figure 2: Interface reconstruction of a thin filament using PLIC method with (a) correct material ordering (b) incorrect material ordering and (c) using our power diagram method

where δ is the volume fraction gradient and \mathbf{x}_c is the centroid of the cell, Ω . A limiter on δ is used to keep $0 \leq f(\mathbf{x}) \leq 1$. The material locations are then calculated as the center of mass of this reconstruction:

$$\mathbf{x}_m = \int_{S(\Omega)} \mathbf{x} \tilde{f}(\mathbf{x}) dx dy \quad (4)$$

To eliminate mesh artifacts on unstructured grids, the domain of integration $S(\Omega)$ is chosen to be the bounding square of the cell with the same centroid as the cell.

For thin filament structures such as those encountered in Rayleigh-Taylor instability calculations, an incorrect material ordering will place material on the wrong side of the filament leading to “flotsam” and “jetsam”. Our material order independent, power diagram method avoids this problem as shown in Figure 2.

Material order problems arise in other contexts as well. When a bubble rises through a fluid to a free surface it may have a thin film covering it. A material order dependent method, with an incorrect ordering may completely destroy the thin

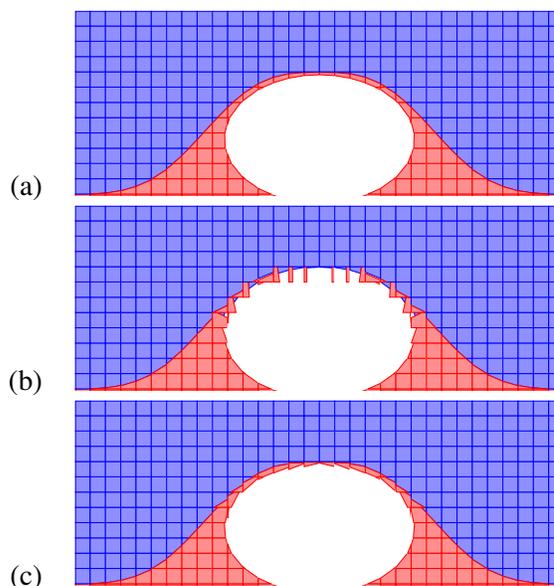


Figure 3: Interface reconstruction of a bubble rising to a free surface using PLIC method with (a) correct material ordering (b) incorrect material ordering and (c) using our power diagram method

film. With no material order dependence, our method correctly locates all of the materials as shown in Figure 3.

Current research is focused on higher order approximations to material centers of mass within a cell and smoothing of the interface resulting from the power diagram based reconstruction.

References

- [1] F. AURENHAMMER. Power diagrams: properties, algorithms and applications. *SIAM J. Computing*, 16(1):78–96, 1987.
- [2] W. J. RIDER AND D. B. KOTHE. Reconstructing volume tracking. *Journal of Computational Physics*, 141:112–152, 1998.

Acknowledgements

This work was carried out under the auspices of the National Nuclear Security Administration of the U.S. Department of Energy at Los Alamos National Laboratory under Contract No. DE-AC52-06NA25396, and supported by the Advanced Simulation and Computing Program.

Los Alamos Report LA-UR-07-0587.