On Dynamics of Influenza

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I. Model (one closed subset)

$N(t)$ - is the total number of individuals in some set at the moment t.
$S(t)$ - susceptibles,
$I(t)$ - infected
$R(t)$- removed (recovered immune component).

$$\dot{I} = \alpha IS - \gamma_D^I I - \gamma_R^I I \tag{1}$$

$$\dot{S} = -\alpha IS + \mu_B^S (S + I) - \gamma_D^S S \tag{2}$$

$$\dot{R} = -\gamma_D^S R + \gamma_R^I I \tag{3}$$

γ_D^I death rate for infected subset
γ_R^I - recovery rate for infected
μ_B^S - birth rate for "healthy" subset S
γ_D^S - death rate for S.
$N(t) = S(t) + I(t) + R(t)$.

$$R(t) = R(0)e^{-\gamma_D^S t} + e^{-\gamma_D^S t} \int_0^t e^{\gamma_D^S z} I(z) dz \tag{4}$$

$$S(t) = N(t) - I(t) - R(t) \tag{5}$$

$$\dot{I} = \frac{\alpha \gamma_D^S}{\mu_B^S - \gamma_D^S} (R_0 e^{-\gamma_d^S t} + e^{-\gamma_D^S t} \int_0^t e^{\gamma_d^S z} \gamma_R^I I(z) dz) I - \frac{(\gamma_D^I + \mu_B^S)}{(\mu_B^S - \gamma_D^S)} I^2 + (\alpha N - \gamma_D^I - \gamma_R^I) I \tag{6}$$

II. Model (interacting subsets)

$N_i(t)$ - is the total number of individuals in some set at the moment t.
$S_i(t)$ - susceptibles,
$I_i(t)$ - infected
$R_i(t)$- removed (recovered immune component).

$$N_i = \sum_{j \neq i} \lambda_{ij} N_j - N_i \sum_{j \neq i} \lambda_{ij} \tag{7}$$

$\dot{\vec{I}} = T\vec{I} - D(\vec{I}\vec{L}^T)$
$\dot{\vec{S}} = T\vec{S} - D(\vec{S}\vec{L}^T)$
$\dot{\vec{R}} = T\vec{R} - D(\vec{R}\vec{L}^T)$

$T = \delta_{ij} + \lambda_{ij}(1 - \delta_{ij})$
$\vec{L}_i = \sum_{j \neq i} \lambda_{ij}$ and $D(A_{ij}) = \vec{A}_{ii}$

$$\dot{\vec{I}} = \alpha D(T\vec{I}_i - D(\vec{I}\vec{L}^T)(\vec{S} - D(\vec{S}\vec{L}^T))^T - \gamma_D^I(T\vec{I} - D(\vec{I}\vec{L}^T)) - \gamma_R^I(T\vec{I} - D(\vec{I}\vec{L}^T)) \tag{8}$$

$$\dot{\vec{S}} = -\alpha D(T\vec{I}_i - D(\vec{I}\vec{L}^T)(\vec{S} - D(\vec{S}\vec{L}^T))^T + \mu_B^S((T\vec{I} - D(\vec{I}\vec{L}^T)) + (T\vec{S} - D(\vec{S}\vec{L}^T))) - \gamma_D^S(T\vec{S} - D(\vec{S}\vec{L}^T)) \tag{9}$$

$$\dot{\vec{R}} = -\gamma_D^S(T\vec{R} - D(\vec{R}\vec{L}^T)) + \gamma_R^I(T\vec{I} - D(\vec{I}\vec{L}^T)) \tag{10}$$

γ_D^I death rate for infected subset
γ_R^I - recovery rate for infected
μ_B^S - birth rate for "healthy" subset S
γ_D^S - death rate for S.

$$\vec{S(t)} = \vec{N(t)} - \vec{I(t)} - \vec{R(t)}.$$

$$\vec{R(t)} = Y(t) * Y(0)^{-1} + \gamma_R^I Y(t) \int_0^t Y(z) D(T\vec{I(z)}_i - D(\vec{I(z)}\vec{L}^T) dz \tag{11}$$

$Y(t)$ solution for $\dot{\vec{R}} = -\gamma_D^S(T\vec{R} - D(\vec{R}\vec{L}^T))$

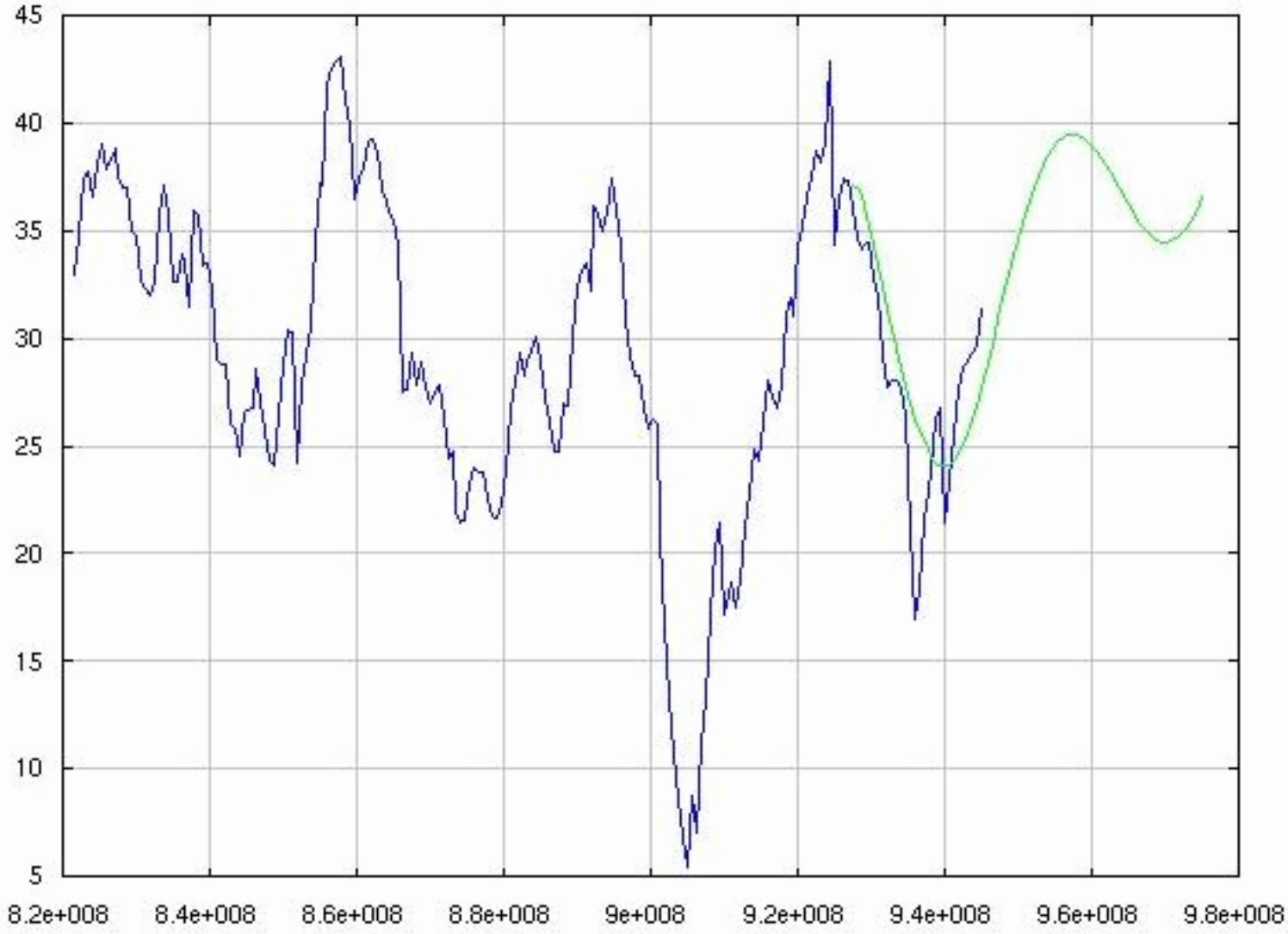
Experimental results.

Comparison of single-set SIR model with actual MMWR CDC data

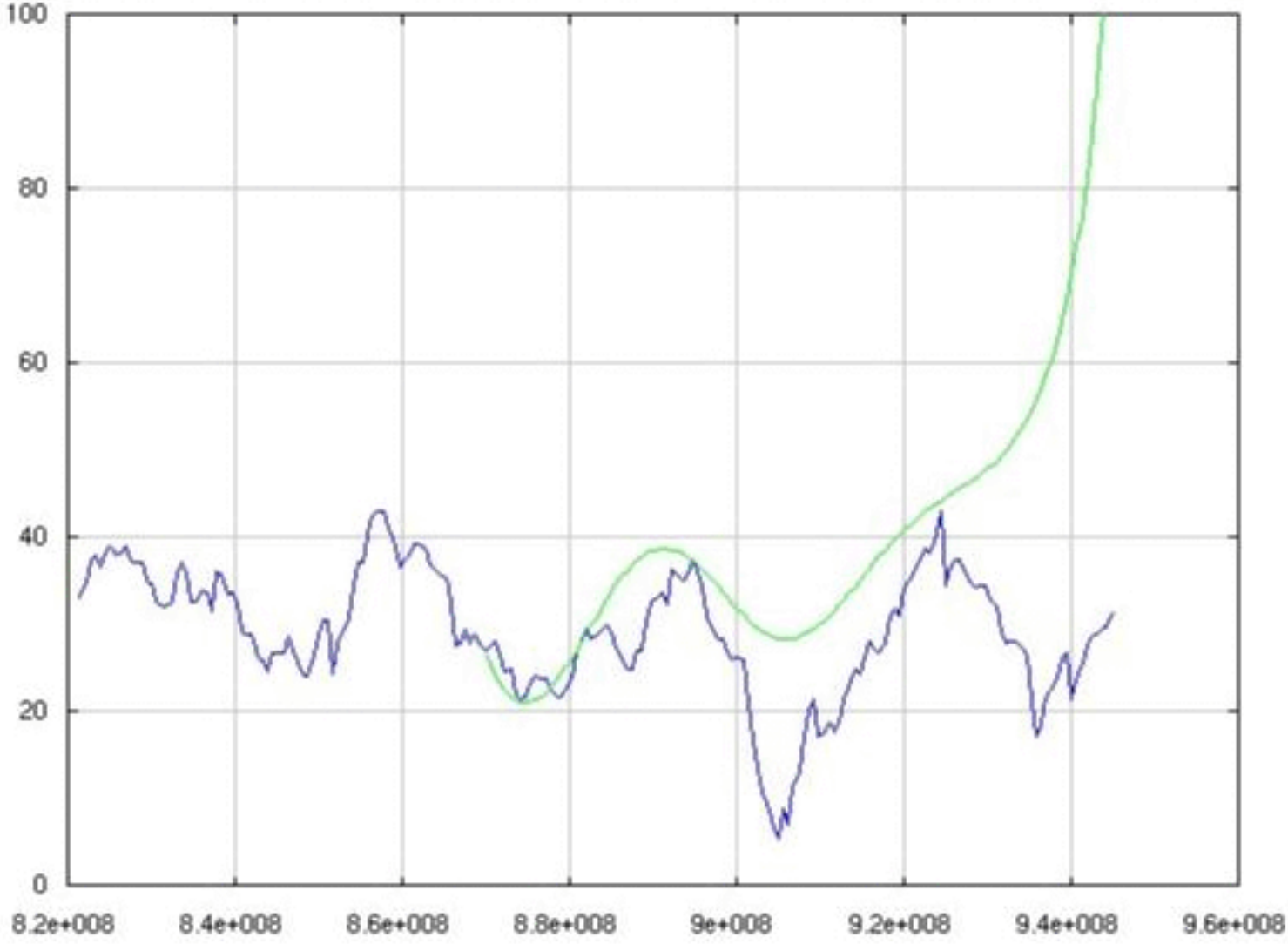
Parameters for SIR system are recovered from the historic time series (blue line). Prediction (green line) is done using different parameters for each city.

Vertical axis - weekly deaths from P&I, horizontal - time (seconds since 01/01/1970).

Chicago



Chicago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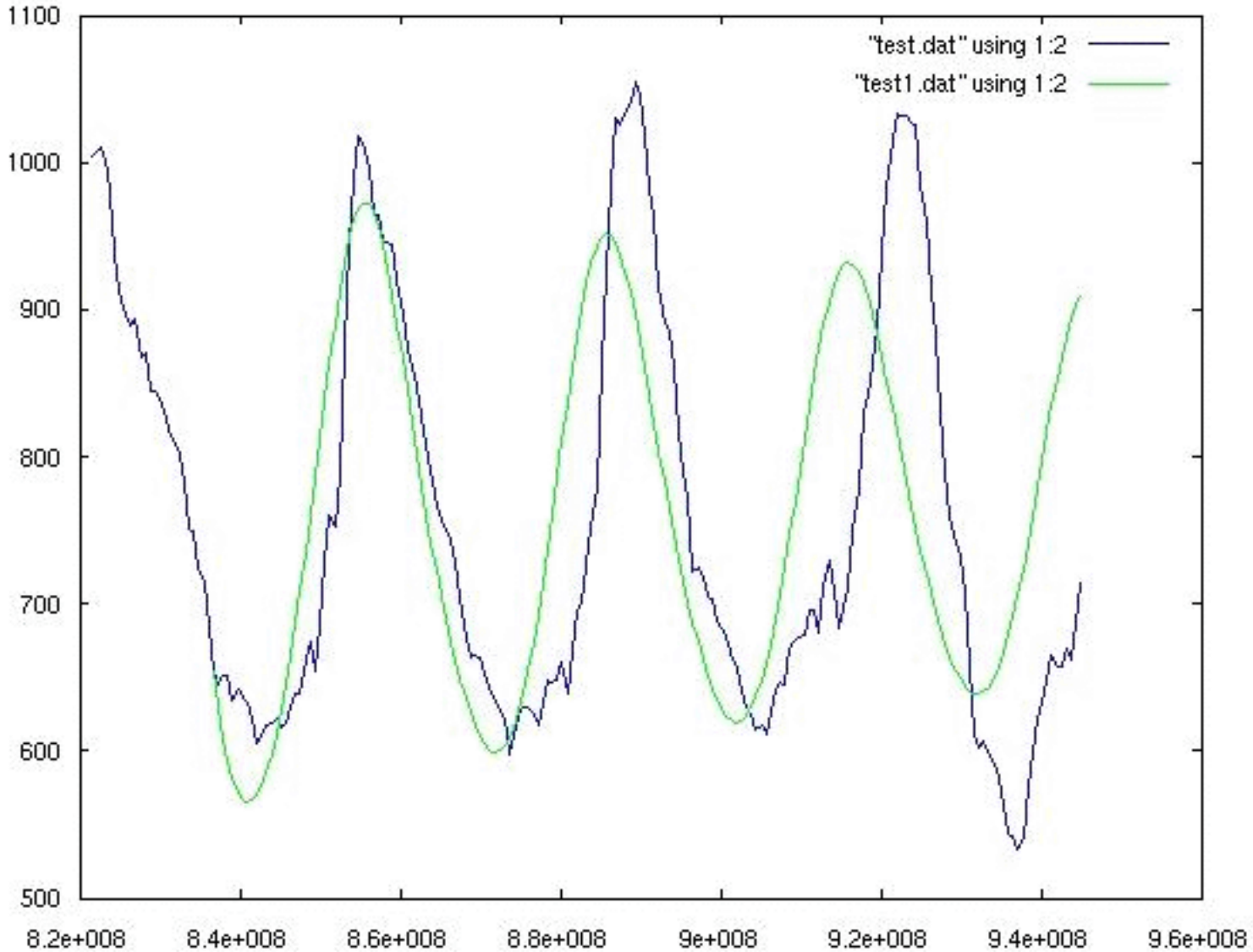
US Total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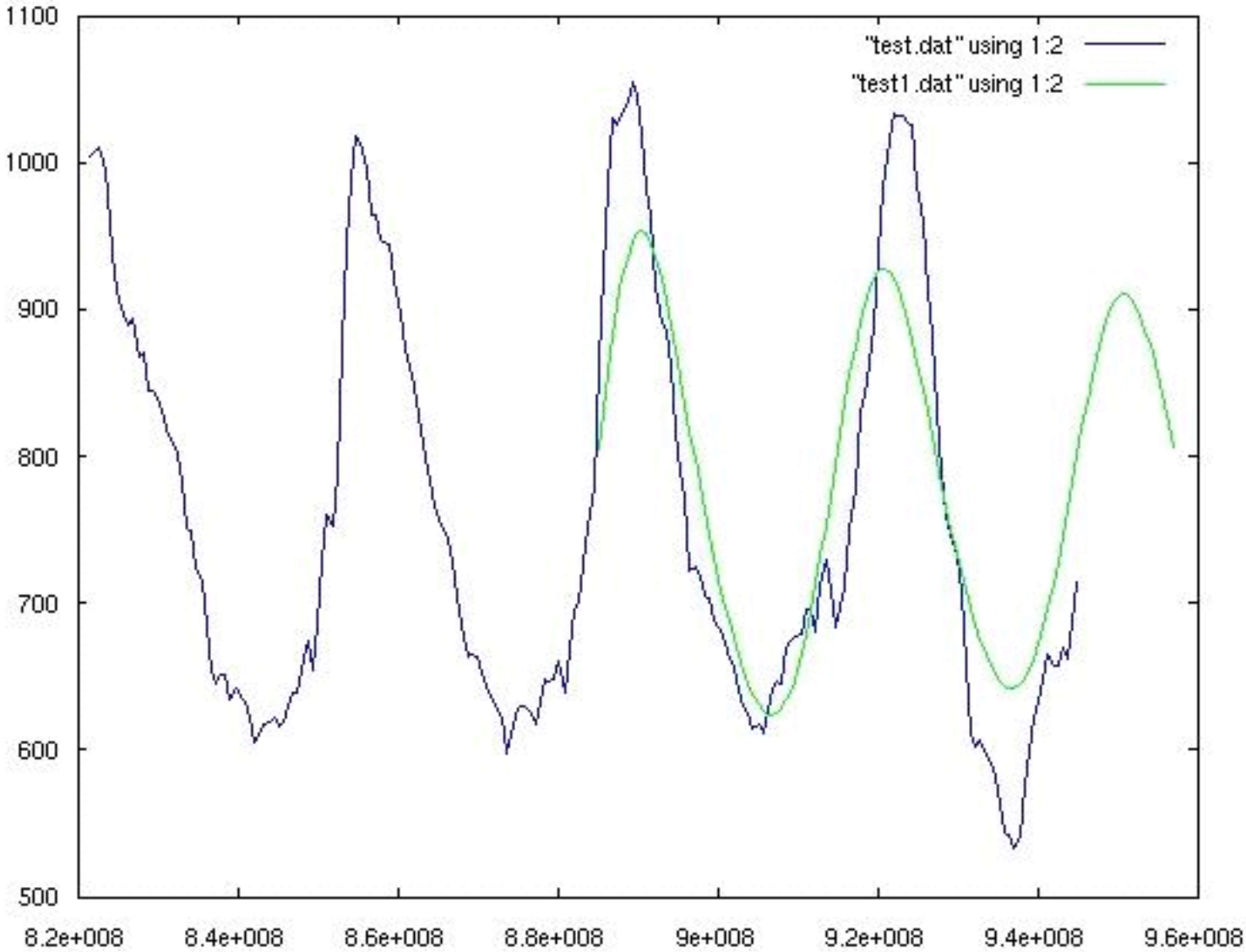
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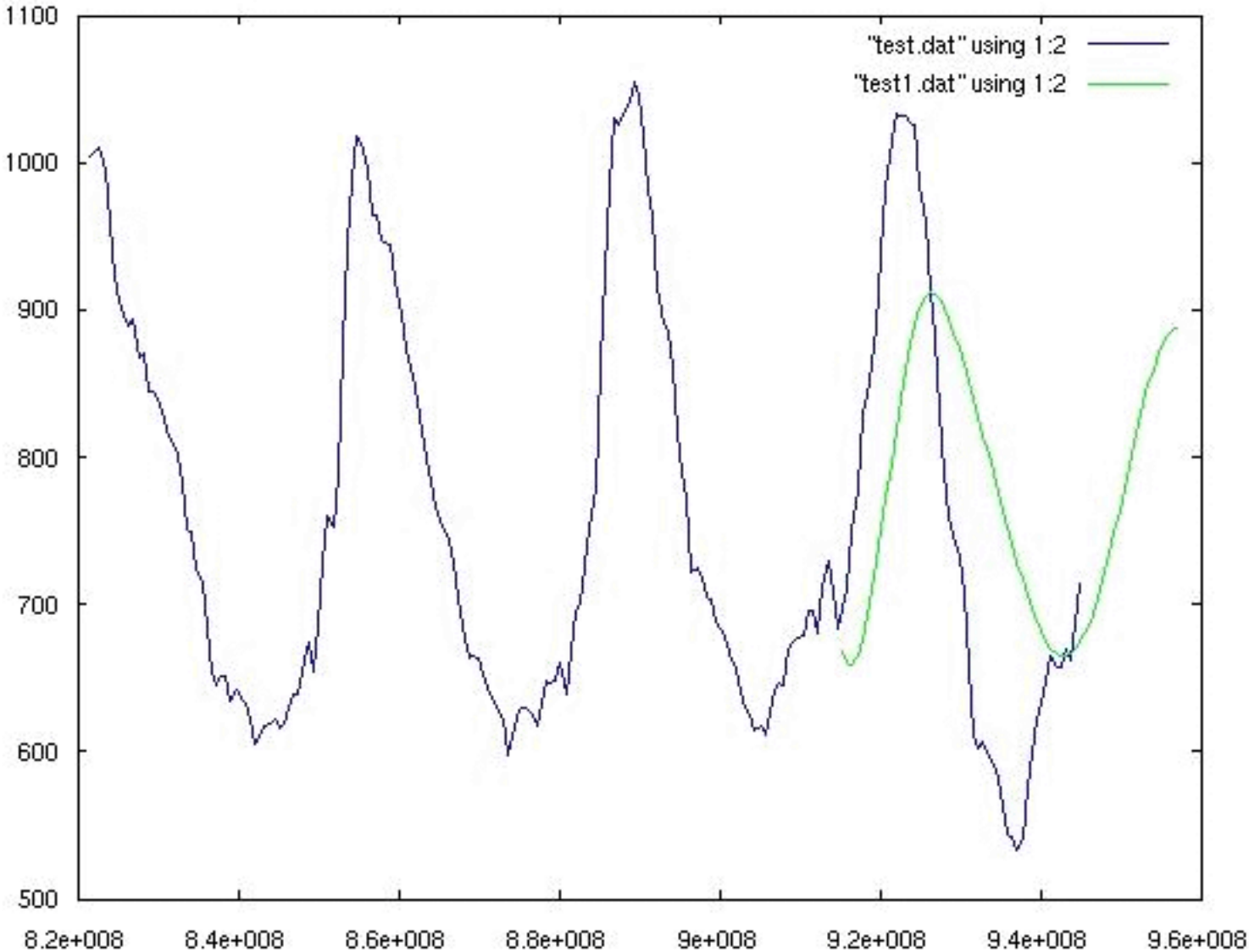
US Total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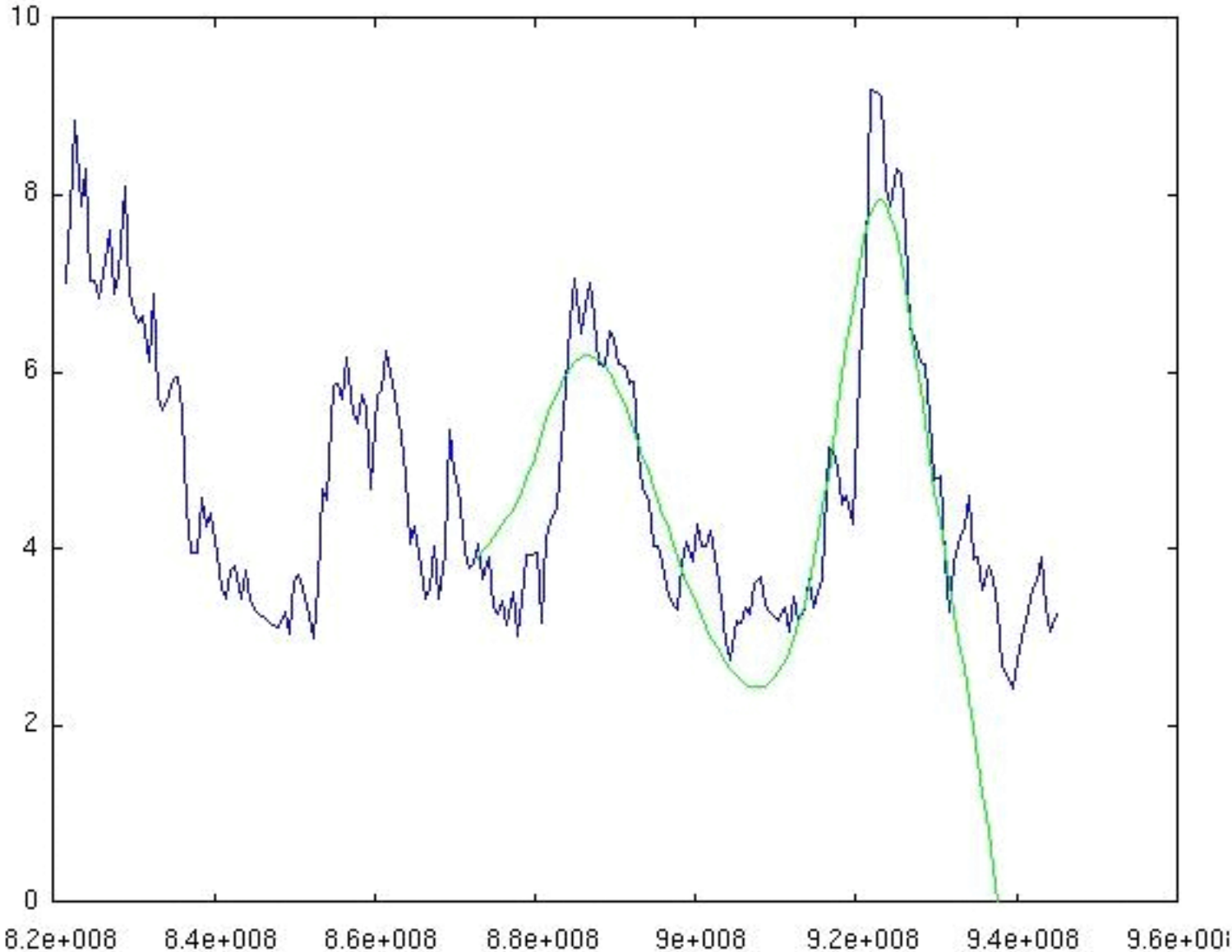
Austin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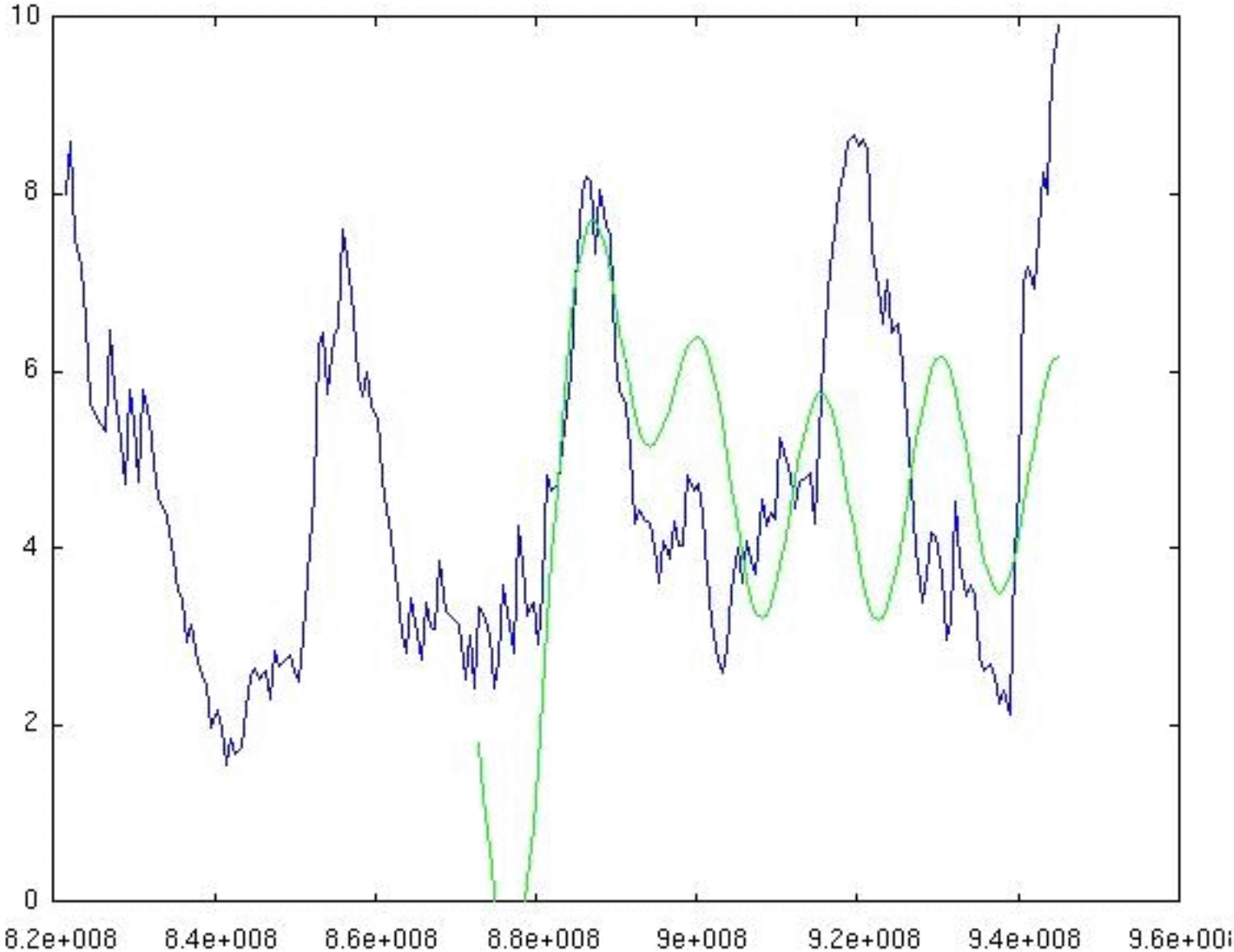
Albuquerque